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DISCUSSIONS

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AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
PAPERS

Standards of Professional Relations and Conduct

BY

DANIEL W. MEAD
Past-President and Hon. M. Am. Soc. C. E.

PREPARED AT THE REQUEST OF
THE BOARD OF DIRECTION OF THE AMERICAN SOCIETY
OF CIVIL ENGINEERS

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STANDARDS OF PROFESSIONAL RELATIONS AND CONDUCT

BY DANIEL W. MEAD, PAST-PRESIDENT AND HON. M. AM. SOC. C. E.[1]

SYNOPSIS

This paper is intended to cover, so far as practicable, the personal and ethical relations of the engineer in all ordinary positions and branches of the profession.

The paper has been written more particularly for the younger men of the profession and with the hope that it may supply them with some information, usually not available in college, as to what their action and conduct should be in their professional relations in practical life. The principles enunciated are those confirmed by the fifty-five years of experience of the writer in the engineering profession. All are principles that the writer believes should not only be acknowledged but actually put into practice in professional service. Their genuine acceptance and practical application would greatly improve the standing of the entire profession, would add to the value of its services, and would greatly strengthen its influence on public opinions. Their application will also accomplish the greatest measure of personal success and personal satisfaction.

The writer first discusses the vital relation of good principles and good conduct to success in life and attempts to analyze the various characteristics necessary for professional success. This is followed by a suggested code of courtesy and personal conduct. Then follow suggested codes for ten departments of engineering activity. The writer recognizes the fact that this paper does not cover, completely, the entire field of engineering. It is believed, however, that a careful consideration of these suggested codes will make obvious to the engineer what his course of conduct should be in almost any field not specifically covered. It is not suggested that any such involved code be adopted as the official code of the Society, but it is hoped that the discussion will be sufficiently broad and complete to determine whether or not the principles outlined meet the approval of the profession. From the paper and its discussion, it is hoped that an unofficial code may be developed sufficiently

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NOTE.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by **May 15, 1940.**

[1] Prof. Emeritus, Hydr. and San. Eng., Univ. of Wisconsin; Cons. Engr., Madison, Wis.

complete so that any engineer in any ordinary position or in any usual line of work may determine the opinion of the profession as to the proper course of action which should be pursued. A Bibliography of the Literature on Ethics and Human Engineering is added as an Appendix.

Introduction

While President of the Society in 1936 the writer pointed out that most engineering ethical codes seemed to apply almost exclusively to the engineer in general practice and not to the more than 90% of the profession who are public or private employees.[2] He suggested that it would be desirable to prepare an unofficial code of ethics which would apply directly to all members of the engineering profession in all its ordinary diverse branches and positions. During 1936 he discussed before many of the local sections and student chapters the question of professional relations and professional ethics and found the subject apparently of considerable interest.

The subject was referred by the Board of Direction to its Committee on Professional Conduct who reported that they believed it undesirable to attempt to formulate an official code which would, if inclusive, become too long and too involved. They advised, and the writer believes correctly, that the official code (29)[3] be kept simplified, as at present, with the understanding that the word "client" be considered to be inclusive of the word "employer" which makes the code of much wider and more general application. They advised, however, that someone be appointed by the Board of Direction to prepare an inclusive paper on this subject as a basis for a wide and full discussion and that such paper after discussion be made readily available to all engineers.

The writer was drafted by the Board of Direction to prepare this paper and has attempted, so far as practicable, to cover all ordinary work of the engineer.

In discussing this subject and prior to attempting to suggest rules of conduct for the engineer, it seems necessary to show how important such principles are to success in the engineering profession. It is self-evident that all men desire success in their undertakings; otherwise, they would not attempt them. The question "What is success?" can hardly be answered in any brief or exact way as the ideals of success vary with each individual. The ideals may be centered around wealth, fame, honors, political preferment, professional standing, home and family relations, or many other objectives too numerous to be cataloged. Usually the ideals are composite and include a fair share of several such objectives.

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[2] "The Engineer and His Code," by Daniel W. Mead, *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 1483.

[3] Numerals in parentheses, thus: (29), refer to corresponding numbers in the Bibliography; see Appendix.

Whatever may be the ideals of the individual, if they are worthy, their attainment requires health, intelligence, courage, native ability, dependability, education and training, experience and ability to apply theory to practice, character, opportunity, hard work, the will to succeed, and personality. (A good personality is the result of acquiring a wide range of habits and abilities for dealing with other people in the manner in which they like to be dealt with (18).) Each of these attributes is partly independent and partly dependent on individual effort and can be acquired, strengthened, and made effective by the exercise of care, perseverance, and energy.

Every one is born with certain physical and mental endowments and capabilities which should be properly developed. Early development is dependent on home influences and environment but the time normally comes when the individual reaches an age where he must assume responsibility for his own future welfare.

Time is one of the most important possessions of every one and its proper expenditure is very important. If time is wasted in the minor pleasures of life instead of being used in the development and improvement of talents and opportunities then only minor professional advancement can be expected.

To learn so to budget one's time that it may be successfully utilized in acquiring the ability to apply the theory and principles of technology and science to professional practice is the soundest preparation for professional accomplishment and advancement. It should be understood that the engineering student becomes an engineer only when he can apply, successfully, theory to professional practice. A reasonable amount of physical exercise and of social intercourse is both desirable and essential for individual development but excess in either study or pleasure is unfortunate. A well-balanced expenditure of time is the desirable aim. The time spent in the office (usually eight hours per day) in drafting, designing, planning, etc., is too short to assure early acquaintance of the young engineer with his job. Work, rest, and other necessary expenditure of time will commonly leave at least four hours of spare time, and these hours are of great importance in determining the speed of advancement of the young engineer and of his ultimate professional success. The budgeting of these often wasted four hours to average not more than one half to recreation and the other half to study that is collateral to his job will gain one full day of advantage every four days over those whose days end when they pass out of the office door. Early acquaintance with the job and possible early advancement may result to the young engineer through the intelligent use of his spare time.

The mental point of view of the individual is a matter of great importance to his success in life. There is a tendency in the scientific teaching of the present time to consider that the possibilities of the individual's progress are limited and that his advancement is circumscribed by his hereditary endowment, by his intelligence quotient (I.Q.), by his glands, by inhibitions, by inferiority complexes, etc., to a very limited field of endeavor beyond which he cannot attain. In the opinion of the writer, this theory omits the most important factor of the individual—his personal energy and will power. If the individual neglects his own personal power of self-control, and will to achieve,

he will float with the tide of affairs and take only that share of life which comes to him naturally. On the other hand, if instead of taking things as they come, he exerts himself through introspection and study, if he determines his own strength, weakness, and defects, and by strenuous personal endeavor attempts to strengthen and improve such weaknesses as he may find in his own character and personality, he can accomplish fine results which otherwise cannot be attained. Defects of character and personality can be overcome by strenuous endeavors of the individual, and by him only. Great accomplishments are possible if the individual has the will to accomplish and is willing to work. Ambition and the will to work toward a desired end will result in high attainments. Genius is but the exercise of infinite pains to accomplish a desired end.

Intelligence and Native Ability.—The writer has always believed that, if one has the ambition and the necessary industry, courage, and determination, any man with fair intelligence and ability can make a reasonable success in any line of work which he desires to enter. The higher the degree of intelligence, the better the chances for greater success, other things being equal. It does not follow that men of the keenest intellect will always make the best engineers, as the ability to apply knowledge and many other factors are of equal importance.

Dependability.—If an investigator considers any engineering position, from that of draftsman in the office or chainman in the field, to the chief or the consulting engineer in the most responsible positions, he will not find one in which dependability is not of equal or greater importance than technical ability and technical skill.

Every engineering enterprise is the combined work of many men and if the best results are to be secured, every one of these men must fulfil his special functions. Weak character is a detriment in all important positions and in all important projects. Every man, ultimately, will be consigned to those places where his character and abilities can be used to the best advantage or where his weaknesses will do the least damage to the project of which he is a part. If he will, every one can train himself to be dependable. Dependability requires not only the necessary technical ability to perform the work on which the individual is employed, but also the characteristics of faithfulness to trusts imposed, honesty of purpose, and loyalty to the service in which the individual is engaged.

The dependable man is assured employment as long as opportunity for his services exists, but the nondependable employee will be discharged as soon as he can be replaced, conveniently, by one in whom dependence can be placed. Lack of dependability and failure to "carry on" in accordance with instructions renders the individual useless in engineering or other business relations.

Technical ability may increase with experience, practice, study, and effort; but lack of character is a fatal defect which, unless remedied, will surely result in failure.

Technical Training and Technical Ability.—It is a truism that, to fill successfully any position, any man must have enough technical ability and technical knowledge and training to perform properly the functions of the position he is to hold. In addition, if he is to attain advancement and a more important

position, his knowledge and training must be extended and increased so that he may be known to be capable of meeting the higher requirements of such a position successfully. For this reason, the college commencement is not the end, but the beginning, of strenuous professional study.

Technical education and technical skill, however, are never rated as more than 30% of the attributes required for maximum success; the other 70% is made up of character, personality, and other moral and mental attributes.

The necessary training and education are not confined to work in an engineering school. Many of the most successful engineers of the past have not had the opportunity for college work. In some way they have had the opportunity to become associated with some form of engineering work and their interest and ambition have become so aroused, and they have worked and studied to such advantage, that many of them have achieved a marked degree of success in the engineering profession. Men with ambition, fair ability, and determination will always find success possible in the profession, with or without the benefits of a college course.

In general, at the present time, the most common method of entering the engineering profession is through the engineering schools. Most students of engineering education will agree that the most important subjects in the curriculum are mathematics and the natural sciences and that the study of professional subjects in any great detail is inadvisable, because all such subjects are changing rapidly and few schools can keep in step with those changes. However, a certain amount of professional study is advisable in engineering education for, through this means it is possible to give the student some idea of the application of theory to practice.

Ability to Apply Principles to Practice.—The ability to apply knowledge to the practical affairs of the profession is a most important matter and one that the college man must acquire if he is to be successful in his work. The practical man who studied as he worked has comparatively little trouble in this respect as his studies have been for a particular purpose. For this reason the writer believes that some knowledge of practical things is of great importance in orienting the studies of the school. When a student has had practical experience, he usually has a better idea of the purpose of his studies and thereafter he often pursues them with much greater vigor and profit to himself for he then recognizes their utility.

Occasionally men of high intellectual attainment, who have been in school continuously, with no contact with practical life, have great difficulty in applying their knowledge to engineering work and therefore fail to attain the degree of success which their intellect would seem to warrant.

The writer believes that the opinions expressed herein are essentially the opinions of most mature men in the profession. The evidence of this is best shown by the investigation of Prof. C. R. Mann,[4] M. Am. Soc. C. E., which was made for the Carnegie Foundation for the Advancement of Teaching. In the spring of 1915 letters were sent to the members of the national technical societies requesting individual opinion on the essential characteristics of an

[4] Address on "Engineering Education," by C. R. Mann, *Proceedings*, Am. Soc. C. E., February 1916, p. 98.

engineer. A report[5] was made in the spring of 1916 as a result of the inquiry, in which the essential factors were summarized into six groups which were numbered in the order of their importance as determined by the frequency of their occurrence in about 1,500 replies. Later a new letter was mailed to the members of the societies asking for further consideration of the six characteristic groups previously outlined. To this letter, 5,441 replies were received which were sufficiently definite for classification. From these letters the most probable values of these groups of qualities were computed with the results shown in Table 1, which seemed to indicate a rather definite ideal of the professional engineer as to the characteristics necessary for the greatest success.

TABLE 1.—CHARACTERISTICS NECESSARY TO SUCCESS IN ENGINEERING

Item No.	Characteristics	Estimate (in Percentages), Based on:	
		1,500 replies	5,441 replies
1	Character, including integrity, responsibility, resourcefulness, and initiative	41.0	24.0
2	Judgment, including common sense, scientific attitude, and perspective of life	17.5	19.5
3	Efficiency, including thoroughness, accuracy, and industry	14.5	16.5
4	Understanding of men, executive ability	14.0	15.0
	Sub-total	(87.0)	(75.0)
5	Knowledge of fundamentals	7.0	15.0
6	Technique of practice and of business	6.0	10.0
	Sub-total	(13.0)	(25.0)
	Total	100.0	100.0

It will be understood that the relative weight assigned to the groups of personal characteristics in Table 1 indicates in a general way only the comparative values of the desirable personal characteristics of successful engineers as expressed by the members of the national engineering societies who answered the letter. All of these characteristics are necessary for success, and any characteristic, like any link in a chain, must be able to stand any strain to which it may be subjected or the character, like the chain under similar circumstances, is useless and hence a failure. Every engineer must be capable of performing, satisfactorily, in the positions he is called upon to serve, and if he lacks the strength of character necessary for such positions it is fatal to his success.

The important matter is the great weight given to characteristics on which dependability and integrity rest as compared with those factors common to educational requirements; and yet those latter factors are absolutely essential to the success of the engineer.

It is evident, therefore, that, in the opinion of experienced engineers, every successful engineer must be a man of integrity and dependability as well as a man of ability. This does not mean that a man cannot run a transit or level, or be a good draftsman even if he is dishonest, but it does mean that such a man cannot occupy, successfully, a high position of trust and responsibility.

Much could be written covering the desirability and the necessity of high ideals, proper conduct, and courtesy in engineering relations, but the writer

[5] "What is an Engineer," by C. R. Mann, *Engineering Record*, Vol. 74, 1916, p. 10.

believes that enough has already been written to bring these facts "home" to any thoughtful man.

Briefly, it follows that, for success, the practicing engineer must have the characteristics of dependability, stability, perseverance, honesty, thoroughness, courage, foresight, enthusiasm, and cooperation; and for the greatest success there must be included a good character, a fine personality, cheerfulness, loyalty, courtesy, tact, care, economy, sincerity, and harmony. These qualities, together with opportunity, are fundamental to the highest engineering success.

Opportunity.—The best engineering student, if located on a desert island, would never be able to make a success in the engineering profession. Opportunity must obtain if any man is to succeed. In normal times many opportunities are open to the engineering student and usually after the opportunity obtains the question of success depends upon the individual "making good" on his first job which is commonly a stepping stone to a better position and to wider opportunities.

Opportunities are sometimes available through relatives, friends, and acquaintances, but as a rule the man must be able to fill the position satisfactorily or he cannot retain it. In applying for a job, good manners, frankness, modesty, and a good personality are favorable factors.

Success (however it may be defined) depends on two essential factors—opportunity and the man. Rarely, success may be phenomenal. A man especially qualified and, perhaps it may be said, especially inspired, seizes an opportunity and meets it so successfully that he arouses public approbation and gains fame, honors, and many additional opportunities which apparently afford an open way to fame and fortune. Such opportunities, however, are the exception and not the rule. Most successful men acquire success only by the long hard road of individual effort accompanied, of course, by utilizing the conditions that obtain by the intelligent application of their education, training, and experience. Most men have attained success in the past and will attain success in the future, in this longer and more arduous manner.

The common idea conveyed by the well-known verses of John J. Ingalls, that opportunity knocks once and only once at every door and if unheeded departs and never returns, is fallacious and pessimistic. Opportunity is almost always present and almost always at hand if the individual has the intelligence to recognize it and the energy to utilize it. The discouraging feature to the young is that, commonly, opportunity presents itself in the disguise of hard work and, therefore, is not always recognized nor received with enthusiasm.

The actual conditions are best stated in the following poem entitled "Opportunity," by Walter Malone:

> They do me wrong who say I come no more
> When once I knock and fail to find you in;
> For every day I stand outside your door
> And bid you wake, and rise to fight and win.
>
> Wail not for precious chances passed away!
> Weep not for golden ages on the wane!
> Each night I burn the records of the day—
> At sunrise every soul is born again!

Dost thou behold thy lost youth all aghast?
    Dost reel from righteous Retribution's blow?
Then turn from blotted archives of the past
    And find the future's pages white as snow.

Art thou a mourner? Rise thee from thy spell;
    Art thou a sinner? Sins may be forgiven;
Each morning gives thee wings to flee from hell,
    Each night a star to guide thy feet to heaven.

Laugh like a boy at splendors that have sped,
    To vanished joys be blind and deaf and dumb;
My judgments seal the dead past with its dead,
    But never bind a moment yet to come.

Though deep in mire, wring not your hands and weep;
    I lend my arm to all who say "I can!"
No shame-faced outcast ever sank so deep
    But yet might rise and be again a man!

Codes of Conduct and Ethics

In preparing the codes of conduct and ethics which follow, the writer has drawn freely from the various codes already adopted by other professions and industries and has included only a few matters which have not been included in other codes. As many matters included in this paper are embodied in many codes, he has not been able to give the original source of each particular principle. In general, each ethical principle is presented without argument as most of them, it is believed, will be accepted as axiomatic. The validity of any principle, questioned by any member of the profession, may be raised in the discussion, which it is hoped will be very general and complete.

Requirements of the Profession

The profession of engineering calls for men with honor, integrity, technical ability, business capacity, and pleasing personalities. The engineer is entrusted with investigating and reporting on the advisability and feasibility of public, semi-public, and commercial works. On the basis of his reports, important works are inaugurated in which large expenditures are involved, and securities sold and purchased. Often the limited means of widows, orphans, and other dependent persons are invested on the basis of his knowledge and good faith, and the results of financial failure of such projects are very serious. The engineer is also entrusted with designing and constructing important public and commercial undertakings in which his honesty of purpose and sincerity of statement must be above suspicion. He acts as a professional adviser, and his advice must be honest and disinterested; he must exercise judicial functions between his clients and contractors, manufacturers, and material supply companies, and his decisions must be fair and impartial. He has moral responsibilities to the public and to his superiors, to his associates and subordinates, which should be exercised with the highest ideals. His work carries with it grave responsibilities to the public which demand his conscientious consideration; and he cannot properly perform his numerous functions unless his ability, conduct, and motives are such as to command the highest respect and confidence.

PRINCIPLES APPLICABLE TO ALL PROFESSIONAL POSITIONS

It may appear to the younger members of the profession that such requirements apply only to the heads of the profession, but it must be remembered that the profession is not static. The business and engineering world is full of examples of men who have started in the humblest positions and who by means of the proper development of their knowledge, intelligence, character, personality, and ability have become the heads of industry and of the professions in the United States. Opportunities are open to every man who has the personality, initiative, ability, and determination to strive for better things; and the hope of advancement should spur every young man to fit himself for the higher positions which soon will become available for new men, as the retirement of those of advanced age, or as changed conditions, make necessary any advancement in the personnel of each industry. The younger man should recognize that his future is involved and he should prepare himself for those higher responsibilities.

The writer has not attempted to do more, in general, than to state ethical principles as this presentation is intended to offer a basis for what it is hoped will entail discussions, by members of the profession, of this very important subject.

Although the following principles are separate under different headings, it should be understood that these principles should be used by all engineers so far as applicable in all their professional relations.

I. THE ENGINEERING STUDENT AND THE YOUNG ENGINEER

The engineering student should understand that he is preparing to enter a profession in which success, in general, is the reward of honest and strenuous effort. Professional success, while always involving opportunity, usually depends largely on the individual himself and on his intelligence, his personality, his ambitions, his courage, his industry, and his determination.

1. The mental discipline necessary to do well the work of a required but distasteful course is of great value, as throughout life one must frequently solve new problems and do things one would prefer not to do. Hence, if the student finds his required course includes subjects which he does not like, nevertheless he should give such subjects sufficient attention, for the ability to do work well whether he likes it or not is an essential factor to success.

2. It should be remembered that the college commencement is not the end of professional study but rather that it is the beginning of special investigation and the acquisition of knowledge of the practical application of the fundamental principles acquired in college. If a man is to succeed in any branch of the profession he must keep abreast of all new developments in that special field. He must be a constant reader of all books and periodicals that deal with the subject. He must (through active participation in the work of technical societies) keep in touch with the men who are doing important work in engineering. In brief, he must continue to be a student for the remainder of his life.

3. The engineer's employers will seldom ask him what he likes to do for it is his business to like the duties assigned to him. Interest in the job at hand

will induce study and investigation as to how it can be best and most economically designed and completed. Only when the engineer feels an interest in his work can be accomplish the best results.

4. Even if the young engineer's immediate work is foreign to that which he intends to follow in later life, a thorough study and understanding of any engineering subject will be found of great advantage in his later years.

5. The last half century has witnessed a great development in engineering literature. This is contained in engineering society publications, in engineering magazines, and in engineering books with which few young engineers are familiar. A knowledge of this literature, especially along the line of the young engineer's specialty, is of great importance. By extensive study the young engineer should utilize this great storehouse of information. "The difference between an educated and an uneducated man is not so much in the greater amount that the educated man knows as in his ability to find out"—Huxley.

Every one must build largely on the experience and precedents of the past. The young engineer should not be rigorously ruled by precedent, however, but he should not ignore it. On the other hand, he should not strive too much for originality. That which is entirely new is likely to be bad in principle or design. When, after thorough study and investigation, a new principle, method, or design is found to be more desirable, the engineer should not hesitate to adopt it; otherwise no improvements in principle, practice, or methods would be possible.

6. Frequently new ideas and new plans are outlined in more or less detail in the engineering press. Such plans seldom should be adopted without careful inquiry as to their success after some years of practical use. Often, apparently good and economical methods or design prove unsatisfactory in practice because their success or failure frequently can be determined only by use. Where new or untried methods are contemplated, the site of such works, preferably, should be visited and inquiry made as to the degree of success which has been encountered in actual usage before such plans are adopted.

7. The young engineer owes it to himself to join such local and national technical societies as are pertinent to the work in which he is engaged; he should attend their meetings whenever practicable, should read the papers at least in so far as they refer to his line of work, present or prospective, and should join in the discussions verbally or in writing whenever he has information which is new or important. Such association results in new friends and acquaintances, and such participation shows his interest in and knowledge of the work and, when new work is proposed, his availability for such future work. Many successful engineers owe their prominence in no small degree to their association with, and participation in, these engineering societies.

8. The young engineer should not take himself or others too seriously; even if he has been near the head of his class, he will find that the application of principles in practice is often a difficult matter, and requires study, inquiry, and sound common sense. The older members of the profession, men of long experience and extensive practice, should be recognized as human and fallible. Their opinions should be taken subject always to careful consideration, and

should not be accepted without careful thought as to their application to the problems at hand. The young engineer should not be a "yes" man; he should agree only when, after careful study, he can conscientiously adopt, as his own, the opinions expressed by others.

9. Some older engineers depend too much upon past experience and attempt to solve, too quickly, new problems on the basis of such experience, and without careful consideration and study. Every problem is a new problem involving some new conditions, and hence requiring careful investigation and thorough study. Any engineer who will not give a new problem due consideration and study should be retired and thus give the man who will do so an opportunity.

10. If possible, every engineer should visit the plants on which he has assisted in the design and construction after it has been in operation for a few years. Preferably, he should visit them without the announcement of his connection with their origin; and he should inquire of the operators as to any faults or troubles in stability, safety, and operation. If the engineer appears as a stranger, he is likely to obtain information which may improve his future designs, although it may not always add to his self-esteem. Many engineers, for lack of contact with the work they have already done, continue for years making the same mistakes in design.

11. The young engineer should do his work as if he were personally responsible for the success of the entire project on which he is engaged. He should do his work as if it were his own and as if his own funds were involved. The personal point of view should always be the basis on which the engineer's work is done. "Will it be safe and economically sound?" "Would I do this work at all, and if so would I do it this way if it were mine?" The questions should always be in the mind of the engineer in charge of any project. It is not always practicable for the engineer, if in a subordinate position, to influence a design so that it will accord with his ideas of economic and sound practice but his study of the fundamental principles and their applicability is essential to his future success.

12. A pleasing personality is of the highest importance. Cultivate, so far as possible, optimistic tendencies; look at the pleasant and attractive side of life; cultivate a friendly attitude toward those whom you meet, and meet them with a smile. They will respond, generally, in a similar manner. This will smooth the way for the adjustment of differences in opinion and the reconciliation of minunderstandings.

13. Consideration for, and a real interest in, others is the foundation of a fine personality, and when accompanied by ability is the surest way to success. In some cases the rough character, heedless of the feelings or the rights of others, may succeed by sheer ability and in spite of—not on account of—his unfortunate personality.

14. The young engineer searching for a job should seek employment under men of recognized standing and ability and of high ideals. Association with men of low ethical standards is sure to affect, seriously, the mental attitude of the young man who is likely to assume such standards are common to the

profession. It is much better to serve with a man of fine reputation and fine character even at a financial sacrifice.

II. A Code of Courtesy and Personal Conduct

A code of courtesy and personal conduct is expressed in the following items (25):

1. The engineer should not take himself or others too seriously. All men are human and subject to error.
2. He should cultivate a sense of humor. It brightens life and eases intercourse. Serious matters, however, should always receive serious consideration and in such affairs levity is out of place.
3. He should not fail to smile and use courtesy to all—equals and subordinates as well as superiors. This includes greetings, tone of voice, and consideration for the personal comfort of all who enter his office, whether visitors or employees.
4. He should not criticize any one adversely in the presence of others.
5. He should not use ungentlemanly language.
6. He should not fail to respect the authority of others, and the personal dignity of subordinates.
7. He should not criticize destructively. He should criticize constructively by suggesting how the error in question may be avoided in future.
8. He should not encourage gossip about fellow workers.
9. He should not countenance animosities or intrigues.
10. He should not fail to give every one credit for good suggestions.
11. He should not fail to be as liberal in praising good performance as he is in censuring bad performance.
12. He should not show partiality or favoritism or injustice in any direction.
13. He should not let personal feelings govern his actions against his better business judgment.
14. He should not belittle any one, not even the humblest worker with whom he is associated.
15. He should not usurp the functions of others or "go over their heads," or undermine the standing of another, or carry tales.
16. He should not carry personal dislike into organization work; if he does not like an associate, he should subordinate his feeling to the general good of the organization.
17. He should not fail to be generous if he finds himself in a position to criticize or upheave another department or individual—the turn of the wheel of business fortune may place him in a similar position.
18. He should not fail, wherever possible, to give the benefit of his thought and information to others in an organization with which he is connected.
19. He should not fail, if he must disagree radically with an associate, or must "fight" him in an organization, to announce the fact frankly to him first, and fight clean and with good nature.
20. He should not fail, if he must resign from an organization, to do so with due regard for his associates whose interests may be affected by his leaving.

21. He should not use merely technical advantage over an associate in an organization. He should guide himself by sound business principles and the spirit of fairness, even under provocation.

22. He should not take advantage of double meanings of words, or unsaid things or unknown facts, or hide behind sophistries or cryptic statements. He should cultivate clearness, letting all know precisely where he stands. Good strategy in business does not mean stealth and secrets; speed, analysis, and publicity are better.

23. He should not make a move involving his associates without first consulting them.

24. He should use correct but simple English in speaking and writing. Proper language is the mark of education, cultivation, and accuracy.

25. When work is to be done, he should not delay. Putting off to the future the things which should be done at once is one of the habits that leads to unsatisfactory service and to failure.

III. Public Relations of the Engineer

It is fundamental that all principles for correct human action and conduct, to be of permanent or even of reasonably lasting character, must be for the best interests of the public; and, although a transitory advantage to a class, a trade, a profession, or an individual may sometimes temporarily be gained by unwise, unethical, political, unlawful, or even by criminal means, such advantage can be only short lived. Ultimately, it will result in a reaction detrimental to the final welfare of the class, trade, profession, or the individual who stoops to the use of such unfair and unethical methods.

1. To do unto others as he would that others should do unto him (26) is the basis of all sound ethical action, and this principle should be applied not only to the immediate personal relations of the individual, but also to the broader relations as they affect associates and the public.

2. The laws of the state and the nation are paramount, and every engineer is bound to support them in principle and practice, and in all reasonable ways to encourage their enforcement.

3. It is the duty of the engineer to satisfy himself to the best of his ability that the enterprises with which he becomes identified are of legitimate character. If, after becoming associated with an enterprise, he finds it is of unsound or questionable character, he should sever his connection with it as soon as practicable. The engineer should engage in no occupation nor undertake any project that is contrary to law or which is inimical to the public welfare.

4. The engineer should endeavor to assist the public and public officials to a fair and correct understanding of engineering matters, to extend the general knowledge of engineering, and to discourage the appearance of untrue, unfair, or exaggerated statements on engineering subjects in the press or elsewhere, especially if those statements may lead to, or are made for the purpose of, inducing the public to participate in unworthy enterprises.

5. In all his actions, the engineer should avoid the appearance of evil; he should not, by word or deed, indicate that he is willing to compromise with either legal or moral standards.

6. The engineer should discourage, in every legitimate manner, the construction of public works that are economically unsound for the community, state, or nation. As engineers in public service are barred, in general, from a frank public expression of opinion concerning such projects, those who are free from the obligations of employment of public bodies are especially obligated to undertake such discussion.

7. Technical discussions and criticisms of engineering subjects should seldom be conducted in the public press but, when possible, before engineering societies or in the technical press. Clear and simple statements of facts and principles that will clarify local problems in the public mind may sometimes be both desirable and essential for the public welfare and for information of the public on local technical matters.

8. It is unprofessional to give an opinion on a subject without being fully informed as to all the essential facts relating thereto and as to the purposes for which information is asked. The opinions should contain a full statement of the condition under which such opinion applies.

9. The engineer in charge of public work or in private work accessible to the public should be mindful of the safety and the convenience of the public. He should use kindness and courtesy in his personal contacts with the public and should furnish such information to inquirers as is consistent with the interest of his employers and reasonably possible without undue interference with his professional duties.

IV. The Personal Relations of the Engineer

To do unto others as he would have them do unto him (26) is the basis of all sound rules of action.

1. To attempt to injure, falsely or maliciously, the prospects, position, or business of another is unethical and unworthy of any reputable engineer.

2. Honorable competition for promotion and opportunity for employment are an essential part of modern democratic civilization; but an attempt to supplant another after definite steps have been taken toward his employment is dishonorable.

3. It is dishonorable to undertake professional work at the price that will not permit the work to be designed properly, supervised thoroughly, and completed satisfactorily.

4. It is unethical for an engineer to compete with another engineer for employment on the basis of professional charges by reducing his usual charges and in this manner attempt to underbid after being informed of the charges named by another.

5. It is unprofessional to accept an appointment while the just claim for compensation or damages—or both—of an engineer previously employed remains unpaid or a settlement is agreed upon unless he or his heirs or assigns neglect to press his claims legally within a reasonable period.

6. The use by an engineer of plans and specifications prepared by manufacturers, supply houses, or patentees, in order to reduce his work and his charges, and thus unfairly compete with other engineers in professional employment, is unfair and unethical.

7. It is unethical for an engineer to create discord among the clients or customers of any individual or corporation furnishing any kind of public or private service, in order to secure the opportunity of furnishing plans for similar service on the pleas of greater satisfaction or reduced cost. This statement should not be considered to preclude an honest report by an engineer employed, without personal solicitation, to report on the possibilities of betterment in public or private service.

8. It is unethical to review the work of an engineer for the same client, except with the knowledge or consent of such engineer, without giving such engineer an opportunity to present his reasons for the plans in question, unless the connection of such engineer with the work has been terminated; and even in that event, the engineer who has prepared the plans under consideration should have the opportunity to explain and defend them.

9. Rule 8 should not be understood to prevent the investigation, with or without notification to the engineer, of any engineering work for which purchase or refinancing is contemplated by another person or persons—or, on work in which improper design or improper or dishonest work is suspected—when such investigation is made for other than the same client or when the investigation is made for parties having a legitimate interest in the economy, safety, and honesty with which such works are constructed, or where such investigation is made in the public interest.

10. It is dishonorable to advertise in self-laudatory language or in any other manner derogatory to the dignity of the profession.

11. It is dishonorable for any engineer to use the advantages of a salaried position, public or private, to compete unfairly with other engineers.

12. It is unprofessional for an engineer to have a direct or indirect interest in any business which will in any way prejudice his judgment in regard to any professional work which he may be called upon to perform or on which he may be engaged.

13. The engineer should take an interest in, and should assist, his fellow engineers by exchange of general information and experience, by instruction and similar aid, through the engineering societies or by other means. He should endeavor to protect all reputable engineers from misrepresentation.

14. The engineer should take care that credit for engineering works or engineering designs is attributed to those who, so far as his knowledge of all matters goes, are the real authors of such work.

15. An engineer in responsible charge of work should not permit non-technical persons to overrule his engineering judgments on purely engineering subjects.

16. When any question of design or construction becomes essential to the safety or to the success of a project the engineer should insist on the correct procedure even if a refusal must result in his resignation. During his fifty-five years of practice, the writer has had occasion several times to lay his job "on the table" before his clients, once before the governor of his state, and at other times before city councils or their committees, boards of directors of corporations, and private clients with the general statement: "This is your job and you

are entitled to have it done any way that you see fit; but you cannot, for the reasons that have been stated, have it done in the way you propose as long as the speaker is your engineer." The question naturally arises, "How often has the writer lost his job on account of taking such a position?" The answer is "Not once." The writer is not advising unnecessary arbitrary dictation as to non-essential details. He would paint his client's house black if after due consideration his client so desired; but he would not knowingly build a poor or unsafe foundation under the house for the sake of any job.

17. It is unprofessional for an engineer to undertake professional work for which he is not qualified by education, training, and experience.

18. As a public official the engineer should realize his own limitations in both training and experience, and should not hesitate to ask and advise consultation on unusual problems on which his own information or experience is incomplete. Elective and also appointive officials often assume that they are endowed with all information and judgment necessary for the fulfilment of their function, and engineers—being human—are not free from this serious defect the result of which is often unnecessary extravagance and wastefulness.

19. Professional work should come to the engineer in general practice without solicitation and on the basis of experience and reputation, as in the case of the medical and legal professions. Active solicitation including lobbying, the use of influence, criticism of competitors, self-laudation, and other sale-agent practices are degrading to the individual and to the profession. Competition by bidding for engineering work should be discouraged. When municipalities receive propositions for engineering work the engineer, if he submits a proposition at all, should do so only on the basis of his regular charges. The offer or payment of commission to secure work, either directly or through an agent, is unethical. It may be illegal and possibly criminal. In general, he should accept no favors that he cannot and does not return in full measure.

20. The conscientious personal point of view of the engineer should always be the basis of his advice in matters of safety in structures, in plans, or in investments; he should always ask himself before he advises a client: "If I were making an investment, would I invest my money in this project?" "If I were building this structure for myself, would I make it any different or any stronger or safer?" "Have I included in my design unnecessary expenses or extravagances?"

If the engineering advice is in accordance with an honest answer to these questions, it is the best advice that the engineer can give; and if it is in error it is an honest error.

V. The Engineer's Relations to Client or Employer

1. The engineer should consider the protection of a client's or an employer's interests, in all honorable ways, his first professional obligation, and, therefore, should avoid every act contrary to this duty. If any other considerations, such as personal or professional obligations or restrictions, may interfere with his meeting the legitimate expectation of a client or employer, the engineer should inform the client or employer of the situation before employment is accepted or

when such conditions arise. This applies equally to the relations of all engineering employees and employers (private clients, corporations, consultants, or government bodies), regardless of salaries received and as long as employment is retained.

2. The engineer should act for his clients or employers only as a faithful trustee or agent, and should accept no remuneration for his work other than his stated salary or charges for services rendered. The engineer should accept no favors, and no collateral employment that might tend to prejudice his position or modify his judgment in the performance of his duties to his clients or his employer.

3. It is unprofessional for an engineer to accept any commission, remuneration, or substantial service or benefits from a contractor, a manufacturer, or from any interested party other than his client or employer. The rejection of favors, or offers of employment, however, should be made pleasantly and without offense. The acceptance of minor courtesies common among friends and acquaintances, such as cigars, theater tickets, meals, etc., may occasionally be sanctioned, but such courtesies should be returned in kind and, as a rule, should leave the credit on the side of the engineer.

4. In accepting service in a professional capacity, unless otherwise specifically agreed, the engineer's implied legal responsibility warrants the assumptions: That he has the required skill and knowledge; that he will use reasonable care and diligence in the exercise of his function; that he will use his best judgment; and that he will be honest and faithful to his trust. It is unprofessional, therefore, for an engineer to guarantee his estimate or contract by bonds or otherwise.

5. The engineer should refuse to undertake work which he believes will result in failure, or which will be unprofitable to a client, without first advising his client as to the nondependability of successful results.

6. It is inadvisable to accept service on questionable projects even when the client is so advised, as the engineer's connection with such a project will undoubtedly cause his name to be connected with a partial or complete failure and may induce investments in the project by others who may believe that his connection with it is an endorsement of its success.

7. Whenever an engineer is employed by a client without definite agreement as to his compensation, he is not warranted in charging more for his services than the fee which he would have charged had such fee been discussed in advance of his employment. It is wise for the engineer to consider carefully the real value of the work he has done for his client and reduce his bill if he finds his charges are higher than are warranted by the real value of the services rendered.

8. An engineer called upon to decide the use of inventions, apparatus, or anything in which he has a financial interest, should make his status in the matter clearly understood by his client or employer before engagement.

9. An engineer in independent practice may be employed by more than one party when the interests of the several parties do not conflict; and it should be understood by all interested that he is not expected to devote his entire time

to the work of one, but is free to accept other engagements. A consulting engineer, permanently retained by a party, should notify others of the affiliation before entering into relations with them, if, in his opinion, the interests might in any way conflict.

10. An engineer should consider it his duty to make every effort to remedy dangerous defects in apparatus or structures or dangerous conditions of operation, and should bring such conditions to the attention of his client or employer.

11. When an engineer undertakes, for others, work in connection with which he may make improvements, inventions, plans, designs, or other records, he should preferably enter into a written agreement with regard to their ownership. In general however (27),

> *A*. A client does not acquire an exclusive right to plans or apparatus made or constructed by a consulting engineer except for the specific case for which they were made.
>
> *B*. If an engineer uses information which is not common knowledge or public property, but which he obtains from a client or employer, the results in the form of plans, designs, or other records shall not be regarded as his property but as the property of his client or employer.
>
> *C*. If an engineer uses only his own knowledge or information or data, which by prior publication or otherwise are public property, and obtains no such data from a client or employer except performance specifications or routine information, then the results in the form of inventions, plans, designs, or other records should be regarded as the property of the engineer, and client or employer should be entitled to their use only in the case for which the engineer was retained.
>
> *D*. All work and results accomplished by the engineer in the form of inventions, plans, designs, or other records, or outside the field for which a client or employer has retained him, should be regarded as the engineer's property.
>
> *E*. Engineering data or information which an engineer obtains from his client or employer, or from other engineers, or which he creates as a result of such information must be considered confidential by the engineer; and although he is justified in using such data or information in his own practice as forming part of his professional experience, its publication without express permission is improper.
>
> *F*. Designs, data, records, and notes made by an employee, and referring to his employer's work, should be regarded as his employer's property.

12. The engineer should regard every position as a part of the foundation for his future, with the understanding that each position (even if it is poorly paid) will be the forerunner of better things if it is filled well and intelligently. A reputation for high ideals, good personality, honesty, dependability, intelligence, initiative, and professional ability is the foundation of professional preferment.

13. It is unethical for an employee while retaining his position with an employer to solicit or to accept other employment to investigate, design, or to

construct work in competition directly or indirectly with his employer or which will engage his time, energies, and interest to the detriment of the regular work for which he is receiving compensation.

14. No employee is warranted in doing, directly or indirectly, any dishonest or illegal thing although such action may be required or expected by the employer. With his principles and his future in mind, he should refuse to take part in any dishonest act and should be ready to quit his job if necessary. When questionable actions or tendencies are evident on the part of his employer, the employee should seek other employment immediately rather than retain his position and be slowly but surely influenced to ignore good ethics and ultimately to comply with unethical or possibly dishonest practices.

15. The employee should begin his work promptly on time and continue it until the regular hour for quitting work. The employee who is faithful to his task is always recognized although this recognition may not always be apparent. The employee who is usually late in arriving and early in departing from his work will likely be the first to be discharged when work is slack.

VI. The Engineer's Relations with His Employees

1. The chief engineer or an engineer employing others in the profession owes to his assistants or employees proper consideration of their personal, financial, and professional welfare and advancement.

2. He should encourage their professional study and initiative, and also high ideals of personal and professional conduct.

3. He should criticize, kindly and sparingly, and commend freely any evidence of good judgment, good suggestions, good work, and good professional conduct.

4. He should be free with professional advice and information, and should encourage his assistants in professional improvement.

5. He should so conduct himself as to command their regard and respect for him both as a man and as an engineer.

6. In his office he should be pleasant and courteous to all subordinates, equals, or visitors in greeting, conversation, and in consideration for personal comfort of all who enter.

7. Correct English should always be used and ungentlemanly language studiously avoided.

8. Other things being equal, promotion should be made on merit only, and preference should be given to employees over outsiders.

9. Partiality, favoritism, and injustice should be avoided studiously.

10. Even the most humble worker in an organization should never be belittled. No one should be criticized adversely before others. The authority of others and the personal dignity of subordinates should be respected.

11. Where criticism is necessary it should be constructive, and the employees should be shown how the error in question can be avoided in the future.

12. He should not fail to give every one credit for good suggestions.

13. When an employee is entrusted with a design, his ideas should be utilized so far as practicable. No design under development should be changed ar-

bitrarily or without sound reasons. Only by reasonable treatment of this kind can the initiative of the office force be retained and its greatest value utilized.

14. If necessary to discharge or deprive workers of authority, he should not fail to do so on the basis of business expediency rather than on personal failure.

15. He should not fail to accept blame, from an outsider, for the mistakes of his organization, and should not "air" inside grievances or differences of opinion.

16. He should not "fire" an employee, but should part with him with regret, delicacy, and personal good wishes; on strict organization principles the employer engineer is to blame for all failure among subordinates.

17. The use of the word "we" instead of "I" in discussing the engineering accomplishments of his office is highly desirable.

VII. Relations of the Engineer with the Contractor

The engineer is not only employed to design and prepare plans and specifications for public and private works, but he is commonly also made arbiter between his client or employer and the contractor who is to furnish materials, machinery, equipment, and labor for the construction of such works. In such position there is sometimes a tendency on the part of the engineer to feel that his principal duty is to his client or employer and too often this feeling results in more or less unfair treatment of the contractor.

1. It is unwise for the engineer to ask contractors to prepare preliminary estimates of cost without reasonable compensation for the time and expense of such estimates. The contractor's knowledge is due to his experience and study, and he should be compensated for his services in the same manner as the engineer would expect to be compensated under similar conditions.

2. In his judicial position as arbiter or umpire between his client or employer and the contractors doing work or furnishing materials or equipment for the works he has in charge, the engineer should act with the highest ideals of fairness and honor.

3. Plans and specifications should be prepared as definite and as specific as possible and should define the exact requirements clearly in every case. They should admit of no double meaning.

4. It is unethical to issue specifications which are unfair or so indefinite that they may be interpreted favorably or unfavorably according to the individual contractor who may be awarded the work.

5. Bids should not be opened until the official date of the letting. Lettings should be conducted with fairness and openness; and, in general, all bids should be opened and read publicly in the presence of all bidders who desire to be present.

6. No change in bids should be allowed after bids are opened and read, and the work should be let to the lowest responsible bidder.

7. In private work no contract or to whom the contract will be refused, if he is the low bidder, should be invited or allowed to bid on the work.

8. The engineer should neither use nor countenance the use of the low bid to secure a reduction in the bidding price of some favored competitor.

9. It is unethical for the engineer to require the contractor, without just compensation, to do work or to furnish material not clearly covered by the plans and specifications, but which may be required to complete the work or to cover defects in the design, plans, or specifications.

10. In matters of dispute, the engineer should always render such decisions as he would feel were justified if he were: (*A*) The owner or employer, or (*B*) the contractor.

VIII. Business Relations

Engineering is not only a profession, but in many cases is related closely to business. In consequence, the ethics of business should be observed by the engineer in all business relations. The business principles enumerated by the U. S. Chamber of Commerce are therefore applicable to the engineer in all his business relations (30):

1. The foundation of business is confidence, which springs from integrity, fair dealing, efficient service, and mutual benefit.

2. The reward of business for service rendered is a fair profit plus a safe reserve commensurate with the risks involved and foresight exercised.

3. Equitable consideration is due in business alike to capital, management, employees, and the public.

4. Knowledge, thorough and specific, and unceasing study of the facts and courses affecting a business enterprise, are essential to a lasting individual success and to efficient service to the public.

5. Permanency and continuity of service are basic aims of business, so that knowledge gained may be fully utilized, confidence established, and efficiency increased.

6. Obligations to itself and society prompt business to strive unceasingly toward continuity of operation, bettering conditions of employment, and increasing the efficiency and opportunities of individual employees.

7. Contracts and undertakings, written or oral, are to be performed in letter and in spirit. Changed conditions do not justify their cancelation without mutual consent.

8. Representation of goods and service should be truthfully made and scrupulously fulfilled.

9. Waste in any form—of capital, labor, services, materials, or natural resources—is intolerable, and constant effort should be made toward its elimination.

10. Excesses of every nature—inflation of credit, over-expansion, over-buying, over-stimulation for sales—which create artificial conditions and produce crises and depressions are condemned.

11. Unfair competition—embracing all acts characterized by bad faith, deception, fraud, or oppression, including commercial bribery—is wasteful, despicable, and a public wrong. Business will rely for success on the excellence of its own service.

12. Controversies will be adjusted, where possible, by voluntary agreement or impartial arbitration.

13. Corporate forms do not absolve from or alter the moral obligations of individuals. Responsibility will be as courageously discharged by those acting in representative capacities as when acting for themselves.

14. Lawful cooperation among business men and in useful business organizations in support of these principles of business conduct is commended.

15. Business should render restrictive legislation unnecessary through conducting itself so as to deserve and inspire public confidence.

IX. The Ethics of Contracting

Many engineers are engaged in contracting either as principals or as assistants to contractors. The ethical principles adopted by the Associated General Contractors of America (31) (A. G. C. A.) are suggested as applicable to the engineer in such relations. In presenting this code, slight changes have been made to make it applicable to all contractors and not only to members of the A. G. C. A.

The working principles by which all contractors should be governed in their relations with client owners and the public, with other agencies of construction, and with members of their own profession, are as follows:

A. Owners and the Public.—Fair and bona fide competition is a fundamental service of the industry to which clients and owners are entitled. Any other method in restriction thereof is a breach of faith toward all contractors and a betrayal of these principles; but the competition cannot serve its legitimate purpose unless it operates under conditions fair alike to owner and to contractor.

Observance of ethical conduct toward the contractor by those who utilize his competitive bidding will be encouraged in proportion as he himself abides by the ethics of fair competition. Only when he respects this code can he reasonably ask others to respect it. Ethical conduct with respect to competitive bidding is defined as follows:

1. Competitive bids preferably should be submitted only when a definite time and place for the opening of all proposals have been fixed, at which all bidders or their representatives are permitted to be present.

2. The contractor's professional knowledge is the result of his training and experience, and if he is called upon for preliminary estimates or appraisals it is proper that he should be paid in the same manner that engineers and architects are paid for similar service.

3. Bidders should neither seek nor accept information concerning a competitor's bid prior to the opening, nor by any method suppress free competition. It is equally improper for the owners to use bids in an effort to induce any contractor to lower his figures.

4. On private work, if all competitive bids are rejected, new bids should not be submitted within sixty days unless warranted by a substantial change either in the work to be performed, or the market, or other basic conditions affecting cost.

5. The amount of a bid should not be altered after the opening except when substantial change is made in the work, or when further bidding on alternate

items is requested. In the event of such change or further bidding, the contractor should bid only on the items specified and should increase or decrease the amount of his bid only in proportion to the change or alternate involved. Any reduction of a bid disproportionate to such change, or the submission of any alternate which in effect produces such a disproportionate reduction, constitutes unfair competition. This shall not be construed as prohibiting the low bidder from decreasing his bid.

6. Contractors should cooperate in advising architects, engineers, and owners with respect to the relative costs of various alternates while plans are being prepared and thus seek to reduce the number of alternates to a nominal maximum.

7. When bids are solicited and received by an owner on a lump-sum basis, no competitor other than the low bidder should solicit the work on a percentage basis, or any other form of cost-plus contract, provided, however, that any competitor shall have the right to accept the work on a percentage basis if tendered him without guaranteed maximum cost or at a guaranteed maximum cost not less than his original bid.

B. Engineering and Architectural Professions.—Local and national cooperation in matters of mutual concern should be the basic policy of all contractors in their relations with the engineering and architectural professions, the purpose of this cooperation being to establish a clear conception of respective functions and responsibilities, to guard against uneconomical or improper practices, and to carry out constructive measures within the industry.

Ethical conduct toward architects and engineers demands the following:

1. Support should be given to all efforts of these professions to maintain and extend high standards of conduct.

2. Contractors should give full credit to the value of the services rendered by the architect and engineer and neither undermine nor disparage their functions or usefulness.

C. Sub-Contractors and Those Who Supply Materials.—The operations of the contractor are made possible through the functioning of those agencies which furnish him with service or products, and in contracting with them he is rightfully obligated by the same principles of honor and fair dealing that he desires should govern the actions toward himself of architects, engineers, and client owners. Ethical conduct with respect to sub-contractor and those who supply materials requires that:

1. Proposals should not be invited from any one who is unqualified to perform the proposed work or to render the proper service, or to whom, in event that his proposal should be the lowest received, the contractor would be unwilling to award the contract.

2. The figures of one competitor shall not be made known to another before the award of the sub-contract; nor should they be used by the contractor to secure a lower proposal from another bidder.

3. The contract should preferably be awarded to the lowest bidder if he is qualified to perform the contract, but if the award is made to another bidder, it should be at the amount of the latter's bid.

4. In no case should the low bidder be led to believe that a lower bid than his has been received.

5. When the contractor has been paid by a client owner for work or material, he should make payment promptly, and in just proportion, to sub-contractors and others.

X. A Personal Code of Conduct and Ethics

The ideals of the individual are so important to his future that every engineer should determine for himself, after mature deliberation, the ideals of conduct and ethics on which he desires to found his future actions. Any code prepared by others can but approximate the ideals even of the writers themselves, and may fall short of covering the different relations and ideals of any individual. It would seem desirable, therefore, for each individual to formulate, in his own mind at least (and preferably in written form), a personal code so that he may review and alter his ideals as his experience seems to require. Such a code was found among the papers of Thomas Van Alstyne, a graduate electrical engineer of Cornell University, after his death which occurred on the job (28). While this code was evidently prepared for his own use, it should be an inspiration to others as well as a lasting memorial to this young man.

> "To respect my country, my profession, and myself. To be honest and fair with my fellow men as I expect them to be with me. To be a loyal citizen of the United States. To speak of it with praise and act always as a trustworthy custodian of its good name. To be a man whose name carries prestige with it wherever it goes.
>
> "To base my expectations of a reward on a solid foundation of service rendered. To be willing to pay the price of success in honest effort. To look upon my work as an opportunity to be seized with joy and to be made the most of, not as a painful drudgery to be reluctantly endured.
>
> "To remember that success lies within my own self and in my own brain, my own ambition and my own courage and determination. To expect difficulties and force my way through them. To turn hard experience into capital for future struggles.
>
> "To believe in my profession heart and soul. To carry an air of optimism in the presence of those I meet. To dispel all temper with cheerfulness, kill doubts with strong conviction, and reduce action with an agreeable personality.
>
> "To make a study of my business. To know my profession in every detail. To mix brains with effort and system in my work. To find time to do every needful thing by not letting time find me doing nothing. To hoard days as a miser does dollars. To make every hour bring me dividends in increased knowledge and healthful recreation. To keep my future unencumbered with debts. To save as well as to earn.
>
> "To cut out expensive amusements until I can afford them. To steer clear of dissipation and guard my health of body and peace of mind as a most precious stock in trade.
>
> "Finally to take a good grip on the joys of life. To play the game like a man. To fight against nothing as hard as my own weakness and endeavor to give it strength. To be a gentleman and a Christian so I may be courteous to man, faithful to friends, and true to God."

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PAPERS

THEORY OF ELASTIC STABILITY APPLIED TO STRUCTURAL DESIGN

By Leon S. Moisseiff,[1] and Frederick Lienhard,[2] Members, Am. Soc. C. E.

Synopsis

The theory of elastic stability is a study of the fundamental laws that govern the behavior of metals in compression and the application of the knowledge derived from such study to the design of structures. A comprehensive understanding of this behavior will enable the engineering profession to establish rules of design in accordance with the fundamental laws and thereby to construct more dependable as well as more economical structures. A greater freedom in the application of metals will result therefrom.

The elements and shapes in which metals are used for structural members have been studied and tested individually, as well as in combined forms. Tests of members subjected to compression have shown that the member as a whole will fail by flexure as a column, or its component parts will eventually wrinkle into waves. The stress at which these waves become visible depends on the material, the proportions of the elements, and the structural composition of the member. Elements that have wrinkled into visible waves can no longer sustain their proportionate share of the load and a small increase will cause failure.

It has been observed that for certain proportions of structural elements the waves become pronounced when the load approaches the yield point of the material. To obtain a "rough-and-ready" rule for the stability of the elements, the complex behavior of the material, heretofore, has been expressed conveniently by a simplified relation of width to thickness of plates or shapes. Such a relation forms the basis for the rules of design of most structural specifications. The very simplicity of the rule, however, confines the application to narrow limits and hampers the design and its economy.

Note.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by **May 15, 1940**.

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Columns are generally composed of plates and shapes. To develop higher unit stresses in columns, low slenderness ratios are required and at the same time the designer is forced to adhere to specified ratios of plate width to plate thickness. The possibility of designing economical compression members is limited thereby. Such column design is exemplified by the towers of the Golden Gate Bridge in California for which carbon steel was used for the greater part to meet the buckling requirements of the web plates. The same limitation confronts the engineer in the design of plate girders to such a degree that long plate girders so proportioned become uneconomical..

It is intended, in this paper, to establish the buckling resistance of plates or webs reinforced by stiffeners and to develop methods of design which will enable the engineer to utilize the higher buckling resistance of the reinforced plates to their full capacity. This will enlarge the possibilities of forms of design and will effect considerable economy in the resulting structures.

It is not the intention to change any of the established rules but rather to propose additional requirements within the framework of the existing specifications. The rules thus established will become necessarily more complicated, but the additional work will be well compensated by the results.

In this paper the behavior of structural carbon steel, silicon steel, and an aluminum alloy of the duralumin type [3] have been studied, rules of design established, and tables and diagrams prepared for them. Following the same reasoning, the paper can be extended to any other metals by the use of the same formulas.

For the application of the theory of elastic stability to compression or bending of plates reinforced by stiffeners, detailed studies were made and simple rules for practical design were established. These rules can be extended to combined compression and bending. Additional diagrams and tables would have to be made for various values of the stability coefficient between the limits of 4 to 24. Such tables, however, would apply to special structures only, as stiffening girders of suspension bridges; these structures will deserve special studies.

Tests on the behavior of steel reinforced by stiffeners have been made, especially in Europe, to verify theoretical investigations in this line. They have been limited to some phases of the problem. To assure a wider application of the type of structures of which the paper treats, comprehensive tests are needed which should verify the theoretical considerations and indicate the best methods of fabrication. The combined results of theory and tests will open a wide field for structural design.

Notation.—The symbols used in this paper are defined where they first appear and are assembled for reference in Appendix II.

CHAPTER 1.—ELASTIC STABILITY

To acquire a full knowledge of the buckling stability of plates and shapes, a comprehensive and thorough study of the behavior of the metals in compression is required. Such knowledge will lead to conclusions which should be

[3] Known commercially as Aluminum 27S–T.

applicable to the design of structures. An approach to an understanding of the complex behavior of metals in compression has been made in this paper through a detailed study of the elastic stability of plates.

General Expression for Elastic Stability

The behavior of plates subject to compression, shear, or a combination of the two, can be expressed in the general form:

$$\sigma_{cr} = \tau_{cr} = k \frac{\pi^2 E \zeta}{12 \left[1 - \left(\frac{1}{m} \right)^2 \right]} \left(\frac{t}{d} \right)^2 \ldots\ldots\ldots\ldots\ldots (1)$$

in which: t = thickness of plate; d = width of plate; $\frac{1}{m}$ = Poisson's ratio; σ_{cr} = critical compressive stress at which buckling will occur; τ_{cr} = critical shear stress at which buckling will occur; E = modulus of elasticity in compression; k = stability coefficient depending on the dimensions and design of the plate, its end conditions, and kind of stress to which the plate is subjected; and ζ = modulus factor, a value by which Young's modulus of elasticity should be multiplied when the stress exceeds the elastic range.

The critical stress can be computed readily if an expression is established for the stability coefficient k and a numerical value for the modulus factor ζ. Eq. 1 also holds for the critical shear stress τ_{cr}, if the value of the modulus of elasticity in shear is accounted for in the coefficient k. This has been done in the paper.

Expressions for Stability Coefficient, k

Plates Subject to Direct Stresses.—The direct stresses to which a plate is subjected may be expressed (41*a*)[4] in the form:

$$\sigma_y = \sigma_0 \left(1 - \alpha \frac{y}{d} \right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

in which: y = distance of stress from edge; σ_0 = unit stress at $y = 0$, the upper or lower edge of the plate; and σ_y = unit stress at distance y from edge.

The variation of stress intensity over the full width of plate can be expressed by the factor α in Eq. 2. If α is 0, the stress is constant over the entire section which is equivalent to uniform compression. If $\alpha = 2$ the plate is in pure bending. The interrelation between the stability coefficient k and the factor α can be expressed approximately by the empirical equation

$$k = \alpha^3 + 3\,\alpha^2 + 4 \ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

To arrive at a numerical value of the stability coefficient k, the critical stress for a given example is first computed by equating the work done by the external and internal forces. The value of the critical stress is then introduced in Eq. 1 in which the coefficient k becomes the unknown. The procedure has been well illustrated by Professor Timoshenko (41). Each example for the desired external loading must be computed for various length-to-width ratios to

[4] Numerals in parentheses, thus (41*a*), refer to corresponding items in the Bibliography, Appendix I.

establish minima values of the critical stress, and thus a minima value for the coefficient k. In order to obtain a general expression for the stress condition, an approximate formula (Eq. 3) has been created which gives minima values for k for various loading conditions.

For uniform compression $\alpha = 0$ and $k = 4$. By introducing $k = 4$ in the fundamental formula, Eq. 1, and assuming ζ to be unity, the well-known Bryan (1) formula is derived. It is

$$\sigma_{cr} = \frac{\pi^2 E}{3\left[1 - \left(\frac{1}{m}\right)^2\right]} \left(\frac{t}{d}\right)^2 \qquad (4)$$

For pure bending $\alpha = 2$ and k increases to 24. It should be noted that the length-to-width ratio of the plate has been omitted in the expression for k in Eq. 3. It was tacitly assumed that the proportions of the plate are such as to make k a minimum. This is true for plate proportions which cause buckling into unrestrained half waves. For uniform compression a minimum value of k occurs in the case of a square plate. To increase k, and with it the critical buckling stress, the length of the plate must be shorter than its width. This is of little practical consideration. Plates forming parts of columns should not be supported transversely at intervals which make the distance between diaphragms less than the width of the plate. A column design with diaphragms spaced less than the width of the plates would be uneconomical. Therefore, the minimum value of the coefficient k should govern the proportioning of the plate. In other words, the influence of the length-to-width ratio of a plate on direct stresses is relatively inessential.

Plates Subject to Shear.—For plates subject to shear, the length-to-width ratio becomes of prime importance. For an infinitely long plate the coefficient k is 5.35; it increases to 9.34 for a square plate. By further decreasing the length-to-width ratio the value of k will be increased rapidly.

Table 1 gives the values of coefficient k for various length-to-width ratios, $\beta = \frac{l}{d}$, in which l is the length and d the depth of plate. Simply supported edges have been assumed for ratios of β greater than 0.50. For values of β less than 0.50, a partly clamped edge condition was assumed.

TABLE 1.—PLATES SUBJECT TO SHEAR; VALUES OF COEFFICIENT k FOR VARIOUS LENGTH-TO-WIDTH RATIOS, β

β	k	β	k	β	k	β	k	β	k	β	k
0.20	171.0	0.28	81.6	0.36	47.6	0.44	31.8	0.60	18.8	1.00	9.34
0.22	139.0	0.30	70.2	0.38	42.6	0.46	29.4	0.70	14.9	1.20	8.12
0.24	114.0	0.32	61.1	0.40	38.4	0.48	27.2	0.80	12.3	1.40	7.38
0.26	95.9	0.34	53.7	0.42	34.9	0.50	25.4	0.90	10.6	1.60	6.90

Plates Subject to Combined Direct Stresses and Shear.—The interaction of direct stresses and shear can be analyzed by separating the two strains and by expressing the effect of the shear on the critical compression.

If the shear to which the plate is subjected is as high as the critical shearing stress, it is evident that the plate cannot sustain simultaneous direct stresses, in addition. As the ratio of the actual shearing stress to its critical value decreases, the plate will retain an increasing resistance for direct stresses. This is expressed numerically in Table 2, in which the shear ratio is the ratio of the actual shear to its critical value and k_d is the percentage of k available for direct stress.

TABLE 2.—REDUCTION OF k FOR BENDING STRESSES DUE TO SIMULTANEOUS ACTION OF SHEAR

Description	Shear Ratios, $\frac{\tau}{\tau_{cr}}$:									
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
Value of k_d........	0.99	0.97	0.94	0.90	0.84	0.76	0.67	0.55	0.39	0

THE MODULUS FACTOR

The modulus factor ζ (Eq. 1) is unity or less. For critical stresses below the proportional limit of the material, ζ is unity; for critical stresses above the proportional limit, ζ will be less than unity. Its magnitude depends on the critical stress and on the elastic deformation of the material. If the member in compression remains isotropic (that is, the change in elastic properties of the material is the same in all directions), the modulus factor becomes the ratio of the modulus of elasticity at the critical stress to its value below the proportional limit, denoted as $\epsilon = \frac{E_x}{E}\cdot$ For the assumption that the elastic properties change, in the direction parallel to the stress, only orthotropically, and remain the same in all other directions, the factor ζ is equal to $\sqrt{\epsilon}$.

For columns as a whole an isotropic change may be assumed to exist also above the proportional limit. A number of tests have shown this assumption to be correct; the factor ζ becomes equal to ϵ. This is not the case, however, for plates subject to compression (41*b*). For plates, it is proposed to take the average of the two extreme possibilities of deformations above the proportional limit—that is,

$$\zeta = 0.5\,(\epsilon + \sqrt{\epsilon}) \qquad (5)$$

The immediate problem is to establish a relation between the critical stress and E_x, or, what amounts to the same, to find the variable modulus of elasticity above the proportional limit of the material in terms of the critical stress.

A study of column strength for various slenderness ratios will lead to the desired relation. Column strength has been established by a great number of tests. These tests indicate an effective buckling length of approximately 75% of the full length of the column. Accepting this buckling length, Euler's formula may be written

$$\sigma_{cr} = \frac{\pi^2 E}{\left(0.75\,\frac{l}{r}\right)^2} \qquad (6)$$

With this assumption the tests show that the proportional limit is reached at a slenderness ratio of approximately 160 for structural steels and approximately 93 for the aluminum alloy tested. For slenderness ratios below these values, the critical stress is less than given by Euler's formula with a constant modulus of elasticity.

Tests have shown, furthermore, that below a certain slenderness ratio the column strength remains practically unchanged and may be taken as equal to the yield point of the material. This slenderness ratio is about 60 for steel and 32 for the aluminum alloy. The column-strength curves connecting these limiting slenderness ratios must be tangent to the Euler curve at their upper limit and pass through the yield point at their lower limit. Tests indicate for the connecting curve a parabola for carbon steel and straight lines for silicon steel and the aluminum alloy.[3] The empirical equations are: For carbon steel

$$\sigma_{cr} = 19{,}390 - 0.7335 \left[(162)^2 - \left(\frac{l}{r}\right)^2 \right] \qquad (7a)$$

for silicon steel

$$\sigma_{cr} = 60{,}050 - 251 \frac{l}{r} \qquad (7b)$$

and for aluminum alloy

$$\sigma_{cr} = 62{,}500 - 447 \frac{l}{r} \qquad (7c)$$

Introducing the critical stress thus computed into the modified Euler formula (Eq. 6), the value of ϵ, or the variable E, can be computed for various critical stresses. The values of ϵ and $0.5\,(\epsilon + \sqrt{\epsilon})$ for various critical stresses are given in Table 3. With this information on hand the two coefficients k

TABLE 3.—VALUES OF $\epsilon \left(= \frac{E_x}{E} \right)$ AND THE MODULUS FACTOR $\zeta\,[= 0.5\,(\epsilon + \sqrt{\epsilon})]$, FOR CRITICAL STRESSES

Critical stress, σ_{cr}, in pounds per square inch	CARBON STEEL		SILICON STEEL		ALUMINUM ALLOY	
	ϵ	ζ	ϵ	ζ	ϵ	ζ
19,000	1.000	1.000	1.000	1.000	1.000	1.000
20,000	0.999	0.999	1.000	1.000	1.000	1.000
21,000	0.993	0.994	0.999	0.999	1.000	1.000
22,000	0.981	0.986	0.994	0.995	0.999	0.999
24,000	0.941	0.956	0.973	0.980	0.985	0.989
26,000	0.881	0.909	0.940	0.955	0.959	0.969
28,000	0.798	0.846	0.897	0.922	0.923	0.942
30,000	0.694	0.764	0.845	0.882	0.878	0.907
32,000	0.569	0.662	0.785	0.836	0.824	0.866
34,000	0.423	0.537	0.720	0.784	0.765	0.820
36,000	....	....	0.650	0.728	0.700	0.768
38,000	....	....	0.576	0.667	0.632	0.713
40,000	....	....	0.502	0.605	0.561	0.655
42,000	....	....	0.427	0.540	0.489	0.594

and ζ can be computed, and it becomes possible to apply the general expression of elastic stability, Eq. 1.

CHAPTER 2.—THICKNESS OF PLATES AND ANGLES OF COMPRESSION MEMBERS AND COMPRESSION FLANGES OF PLATE GIRDERS

The allowable ratio of plate width to plate thickness depends on the unit stress and the factor of safety desired against local buckling. It is deemed sufficient to design against local buckling with a factor of safety of 2; the critical stress, therefore, is twice the allowable stress.

The width-to-thickness ratios $\frac{d}{t}$ can be determined by the use of the fundamental equation of elastic stability, Eq. 1. Introducing $\sigma_{cr} = 2\,\sigma$ and the numerical value for $\frac{\pi^2 E}{12\left[1 - \left(\frac{1}{m}\right)^2\right]}$ into Eq. 1, the ratio $\frac{d}{t}$ becomes—

For steel:

$$\frac{d}{t} = 3{,}620\sqrt{\frac{k\,\zeta}{\sigma}} \qquad \text{(8a)}$$

and for the aluminum alloy:

$$\frac{d}{t} = 2{,}180\sqrt{\frac{k\,\zeta}{\sigma}} \qquad \text{(8b)}$$

In Eqs. (8) the stability coefficient k depends on the dimensions and end conditions of the plate. For simply supported plates the minimum value for k is 4 and the width-to-thickness ratios become—

For steel:

$$\frac{d}{t} = 7{,}240\sqrt{\frac{\zeta}{\sigma}} \qquad \text{(9a)}$$

and for the aluminum alloy:

$$\frac{d}{t} = 4{,}360\sqrt{\frac{\zeta}{\sigma}} \qquad \text{(9b)}$$

Introducing ζ for various unit stresses σ, the allowable width-to-thickness ratios are given in Table 4(*a*). The basic allowable plate slenderness is 51.7, 39.8, and 30.3 for carbon steel, silicon steel, and aluminum alloy, respectively, as given in Table 4; or, in round figures, 50, 40, and 30, respectively.

To determine the highest permissible plate slenderness, it has been customary to place the critical stress of the plates forming parts of columns equal to the yield point of the material and to use Bryan's formula (Eq. 4) with a correction factor of 0.75:

$$\sigma_p = 0.75\,\frac{\pi^2 E}{3\left[1 - \left(\frac{1}{m}\right)^2\right]}\left(\frac{t}{d}\right)^2 \qquad \text{(10)}$$

For carbon steel with a yield point of 36,000 lb per sq in., Eq. 10 gives a ratio of $\frac{d}{t} = 46.7$, and, for silicon steel with a yield point of 45,000, a ratio of 41.8. This method of computing the permissible plate slenderness is somewhat arbi-

TABLE 4.—WIDTH-TO-THICKNESS RATIOS, $\frac{d}{t}$

Material	Ratios for the Following Values of the Allowable Stress σ, in Kips per Square Inch										
	(a) Column Plates										
	10	11	12	13	14	15	16	17	18	19	20
Carbon steel	72.4	68.5	64.6	60.5	56.3	51.7	....	....	....	....	....
Silicon steel	72.4	68.9	65.4	62.1	58.8	55.5	52.3	49.2	46.0	42.9	39.8
Aluminum alloy	43.6	41.6	39.6	37.6	35.8	33.9	32.1	30.3	....	....	....
Material	(b) Outstanding Parts of Columns										
	10	11	12	13	14	15	16	17	18	19	20
Carbon steel	24.0	22.7	21.5	20.1	18.6	17.1	....	....	....	....	....
Silicon steel	24.0	22.8	21.6	20.6	19.5	18.4	17.3	16.3	15.3	14.2	13.2
Aluminum alloy	14.0	13.3	12.7	12.1	11.5	10.9	10.3	9.7	....	....	....
Material	(c) Beam and Girder Flanges										
	14	15	16	17	18	19	20	21	22	23	24
Carbon Steel:											
Rolled or riveted	46.6	42.8	39.0	35.2	31.5	....	....	....	....	....	....
Welded	42.0	38.6	35.2	31.7	28.3	....	....	....	....	....	....
Silicon Steel:											
Rolled or riveted	48.7	46.0	43.4	40.7	38.2	35.5	33.0	30.7	28.3	26.0	23.6
Welded	43.9	41.4	39.0	36.7	34.4	32.0	29.8	27.7	25.5	23.4	21.2
Aluminum Alloy:											
Rolled or riveted	28.7	27.2	25.8	24.3	22.9	21.4	20.0	18.6	....	....	....

trary; but, as seen, the results do not considerably deviate from the more refined analysis.

Eq. 1 can also be used to determine the buckling strength of outstanding parts of columns. These parts are held at one edge only. For this condition the value of the stability coefficient k is considerably smaller compared to its value for plates supported along two edges. The value of k varies between 0.43 and 1.28 for steel and between 0.41 and 1.25 for the aluminum alloy. The lower values are for a simply supported edge (that is, the outstanding part can rotate freely); the higher values are for a fixed edge. For an elastically built-in edge the value of k is between the foregoing limits. To allow for initial curvature the minimum values of k will be taken. The ratios $\frac{d}{t}$ then become—

For steel:

$$\frac{d}{t} = 2{,}400 \sqrt{\frac{\zeta}{\sigma}} \qquad (11a)$$

and for the aluminum alloy:

$$\frac{d}{t} = 1{,}400 \sqrt{\frac{\zeta}{\sigma}} \qquad (11b)$$

For various values of σ the allowable width-to-thickness ratios are given in

Table 4(*b*). Fillets increase the local buckling stability of the outstanding legs of angles. It was found that the width of the outstanding leg can be reduced by twice the thickness and this width used for determining the allowable $\frac{d}{t}$-ratio. Eq. 1 may also be applied to study the local buckling behavior of the compression flanges of rolled beams and plate girders.

The buckling behavior of compression flanges between lateral supports is vastly different from that of columns subject to axial stress. In columns, the entire section is in compression, and the sectional properties fully define the buckling criterion. In beams, half the section is in tension. This has a great stabilizing effect on the buckling of the compression flanges. The allowable stress in beams will be governed, in all practical cases, by the buckling of the compression flanges. The buckling resistance is increased materially by that part of the section that is in tension; the allowable stress, therefore, can be taken higher than is permitted for columns to result in equal factors of safety. This is recognized in prevailing specifications.

It is evident that the compression flanges must be designed also to assure local buckling stability for the increased allowable stresses. A distinction must be made between rolled, riveted, and welded beams.

Fillets of rolled beams increase considerably the local buckling strength of the outstanding flanges. It was found that the width-to-thickness ratio of the outstanding flange can be increased by approximately 25%. Considering the full width of the compression flange, this ratio must be doubled. Solving Eq. 1 for these conditions, the allowable width-to-thickness ratio of compression flanges of rolled beams will be—

For steel:

$$\frac{d}{t} = 6{,}000\sqrt{\frac{\zeta}{\sigma}} \qquad \text{(12a)}$$

and for the aluminum alloy:

$$\frac{d}{t} = 3{,}500\sqrt{\frac{\zeta}{\sigma}} \qquad \text{(12b)}$$

in which d = depth of flange and t = thickness of flange.

For riveted plate girders the same expression may be applied. If the flanges consist of angles, only the fillets will stiffen the outstanding leg. If the flanges consist of riveted angles and cover plates, the actual compressive stress is considerably less than the computed stress, which is based on the net section. In Eqs. 8, 9, 11, and 12, σ is the unit stress on the net section and ζ the corresponding modulus factor. Adhering to the allowable unit stresses of prevailing specifications, the maintenance of a factor of safety of two is not possible in every case because the doubled values of the allowable stresses exceed the yield stress, beyond which ζ is theoretically zero. Factors of safety of two are maintained up to a stress of 15,000 lb per sq in. for carbon steel and 18,000 lb per sq in. for silicon steel. From these values the factors of safety gradually decrease to 1.92 and 1.85 for the maximum allowed stresses of 18,000 and 24,000 lb per sq in. for carbon and silicon steel, respectively. For the aluminum alloy, a factor of safety of two is maintained for all stresses.

In welded girders the allowable unit stress is fully realized. Although the flanges may be sheared from rolled beams, the fillets may not (in all cases) have the full effect of the rolled section. Therefore, a reduction in the allowable width-to-thickness ratio should be made. It is proposed to use

$$\frac{d}{t} = 5{,}400 \sqrt{\frac{\zeta}{\sigma}} \qquad (13)$$

Eqs. 12 and 13 have been used in preparation of Table 4(c).

CHAPTER 3.—DESIGN OF WEBS FOR PLATE GIRDERS

The buckling behavior of the webs of plate girders is more complicated than that of plates forming parts of columns. In the latter case uniform compression is the rule; in the former it is the exception. Plates in columns form a main portion of the cross-sectional area, and their buckling will cause failure of the member. It is mainly for this reason that the critical buckling stress was placed rather close to the yield point. In plate girders the webs contribute little area to the compression flange. The buckling of the web causes its proportionate share of compression to be transferred to the flange and, since this total stress is small, it will cause a correspondingly small overstress in the compression flange. Furthermore, the buckled web adjusts itself to sustain the shear by tension and transmit it to the vertical stiffener. Buckling of the web of plate girders, therefore, will not cause failure of the structure, and for these reasons a lower factor of safety against buckling of the web is justified.

Factors of safety of 1.5 for compression in flexure or shear and of 1.4 for the combined action of compressive flexure and shear against buckling of webs of plate girders were adopted. These factors of safety are considered adequate and equivalent to a factor of safety of 2 against buckling of plates forming parts of columns.

The buckling strength of the web depends on the magnitude of the axial stress at the toes of the flange angles and the amount of the average shear acting in combination with compression. The problem becomes rather complex and the allowable width-to-thickness ratio of the web (which, herein, is conveniently called "web slenderness") cannot be divorced from the magnitude of axial stress and shear. The effect of these will be discussed in more detail in Chapter 4. A simple example of pure bending will be stated in the present chapter, however.

The stress condition of pure bending prevails at the center of a simply supported plate girder symmetrically loaded where the shear is practically zero; the buckling coefficient k thus has the full value of 24. The greatest allowable bending stress in tension is 18,000 lb per sq in. for carbon steel, 24,000 lb per sq in. for silicon steel, and 21,000 lb per sq in. for the aluminum alloy. The corresponding tension at the toe of the flange angles for average girders is 15,000, 20,000, and 17,000 lb per sq in., respectively. These stresses should also be applied for compression at the toe of the flange angles to be in accord with prevailing specifications which require the same net section for the tension as well as the compression side. The allowable critical buckling

stresses, with a factor of safety of 1.5, therefore, will become 22,500 lb per sq in. for carbon steel, 30,000 lb per sq in. for silicon steel, and 25,500 lb per sq in. for the aluminum alloy. The corresponding values of ζ from Table 3 are 0.98, 0.88, and 0.98, respectively. For these values Eq. 1 results in web slenderness values of 165, 136, and 94. It will be noted that the resulting values are in general accord with standard specifications.

CHAPTER 4.—PLATES IN COLUMNS AND WEBS OF GIRDERS REINFORCED BY STIFFENERS

The principal function of stiffeners consists in increasing the buckling resistance of the plates to which they are attached. The stiffeners divide the plates or webs into panels, and it is evident that their economical usefulness demands such proportioning that the critical stress of the entire structure is equal to the critical stress of the most stressed panel. The dimensioning of the stiffeners must be such that they form nodal lines at their locations when the critical stress is reached.

To increase the size of the stiffeners above the minimum requirements will not increase the critical resistance of the structure because it cannot be stronger than its weakest panel. If rules could be established for the size of the stiffeners to assure equivalent buckling stability, each individual panel could be treated as a unit. For such units equations and tables have already been established in the previous discussion of the buckling stability of plates (Chapter 2).

Plates forming parts of columns, and reinforced by equidistant stiffeners, permit an extreme plate slenderness of $\frac{d}{t}(N+1)$, in which $\frac{d}{t}$ is the basic ratio 50, 40, and 30 for carbon steel, silicon steel, and the aluminum alloy, respectively, and N is the number of stiffeners. The basic web slenderness of plate girders can be increased in a similar manner by the placing of longitudinal stiffeners at proper locations.

A mathematical analysis of the integrated buckling stability of plates or webs reinforced by stiffeners is rather involved. A study of the available literature will show the heavy mathematical "artillery" necessary to arrive at an approximate solution for simple assumptions.

It may be stated that the size of the stiffeners depends generally on the thickness of the plate they must stiffen; increasing the thickness of plate demands increasing the size of stiffeners. Axial load controls the dimensioning of stiffeners in the direction of its action, whereas stiffeners normal to the axial load are governed by shear.

General Theory of Longitudinal Stiffeners in Plates of Columns and Horizontal Stiffeners in Webs of Plate Girders

The buckling strength of the individual panels into which the stiffeners divide the plate or web and of the integrated structure is expressed by Eq. 1. Knowing the dimensions of each panel and the stress conditions to which each is subjected, its critical stress can be computed. This stress should also be the critical stress of the reinforced plate or web. Since the dimensions and

proportions of the latter are known, the equivalent stability coefficient k_e of the reinforced structure can be computed numerically from Eq. 1 as applied to the reinforced structure. The procedure, therefore, is to establish an expression for the equivalent coefficient k_e for the reinforced plate or web which will result in the same critical stresses as computed for the individual panels.

The rigidity of a plate R_p is

$$R_p = \frac{E\, t^3\, d}{12\left[1 - \left(\frac{1}{m}\right)^2\right]} \qquad \text{(14a)}$$

and the rigidity of the stiffener is

$$R_s = E\, I_s \qquad \text{(14b)}$$

in which I_s = moment of inertia of the stiffener. For a stiffener on one side of the plate only, the value of I_s is to be taken along the inner face of the plate; for stiffeners on both sides, I_s is to be taken about the center of the plate. The ratio δ_s of the area of stiffener to the area of plate is

$$\delta_s = \frac{A_s}{t\, d} \qquad \text{(15a)}$$

and the ratio γ_s of rigidity of stiffener to rigidity of plate is

$$\gamma_s = \frac{12\left[1 - \left(\frac{1}{m}\right)^2\right]}{t^3\, d}\, I_s \qquad \text{(15b)}$$

The buckling strength of each panel is expressed by Eq. 1 in which d is now the width of the panel. The coefficient k can be computed for any known stress condition, as shown in Chapter 1. The buckling strength of the reinforced plate or web is equally expressed by Eq. 1 in which d now becomes the total width of the member. Since both critical stresses must be the same, a numerical value for the equivalent coefficient k_e can be computed as follows:

$$k_e = k' \left(\frac{d'}{d_s}\right)^2 \qquad \text{(16)}$$

in which k' = coefficient for the individual panel, d' = width of reinforced structure, and d_s = width of individual panel.

Coefficient k' is only a function of the panel dimensions and stress conditions. The equivalent coefficient is not only dependent on the dimensions of the member and the stress conditions, but also becomes a function of the ratios δ_s and γ_s. A general approximate value is:

$$k_e = \frac{k_0}{4}\left\{\frac{(1 + \beta^2)^2 + 2\,\Sigma\gamma_s \sin^2 (\pi\, c_s)}{\beta^2\,[1 + 2\,\Sigma\delta_s \sin^2 (\pi\, c_s)]}\right\} \qquad \text{(17)}$$

in which k_0 = stability coefficient for the unreinforced plate, β = ratio of $\frac{l}{d}$, l = length of plate between transverse stiffeners, and c_s = ratio of distance of stiffener from edge of plate to width of plate. With $r^2 = \frac{I_s}{A_s}$ and $\delta_s = \frac{A_s}{t\, d}$,

the value for γ_s can be written

$$\gamma_s = 12\left[1 - \left(\frac{1}{m}\right)^2\right]\delta_s\left(\frac{r}{t}\right)^2 \quad \text{(18)}$$

Substituting in the expression for k and solving for $\left(\frac{r}{t}\right)^2$,

$$\left(\frac{r}{t}\right)^2 = \frac{4\dfrac{k_e}{k_0}\beta^2\left[1 + 2\,\Sigma\delta_s \sin^2(\pi\, c_s)\right] - (1 + \beta^2)^2}{24\left[1 - \left(\dfrac{1}{m}\right)^2\right]\Sigma\delta_s \sin^2(\pi\, c_s)} \quad \text{(19)}$$

For maxima values of $\left(\frac{r}{t}\right)^2$ the differential $\dfrac{\partial\left(\frac{r}{t}\right)^2}{\partial\beta}$ must be zero; β then becomes:

$$\beta = \left\{2\frac{k_e}{k_0}\left[1 + 2\,\Sigma\delta_s \sin^2(\pi\, c_s)\right] - 1\right\}^{0.5} \quad \text{(20)}$$

It is seen from Eq. 19 that the value of $\frac{r}{t}$ depends not only on the ratio β and the location and the number of stiffeners, but also on the value of δ_s which is the ratio of stiffener area to area of plate. Since the ratio δ_s affects both sides of the equation, an explicit form for $\frac{r}{t}$ can be solved only by introducing numerical values for the ratio δ_s. In evaluating this ratio it should be noted that, with decreasing values, the expression for $\frac{r}{t}$ in Eq. 19 increases. To establish requirements for the ratio $\frac{r}{t}$, such stiffeners should be assumed in the computations which give the largest radius of gyration with the minimum amount of section.

In these computations the smallest possible values for the ratio δ_s were used. The smallest values are obtained by using stiffeners which give the greatest radius of gyration about the face of the plate or web with the least amount of sectional area. This limiting relation is termed here "highest stiffener efficiency." The relation between the radius of gyration and the area of stiffener is shown in Fig. 1. These curves condense the results of a large number of computations using available rolled shapes. Bulb angles were found to be most efficient for riveted work; bars should be used for small welded sections; and tees with the stem welded to the plate should be used for large welded sections.

Should stiffeners be used which are not as efficient as shown by the curves in Fig. 1, the ratio of δ_s will decrease accordingly. The requirements for the size of stiffeners are based on minima values of δ_s. For values of δ_s above the minimum, the required radius of gyration is slightly smaller. This reduction in the required radius of gyration is fairly negligible. It is advisable to use minima values for δ_s. This becomes clear in the application to practical cases.

In columns the stiffeners sustain their proportional share of compression and assist in stiffening the plate. No waste of material is caused by using a heavier stiffener of low efficiency. In plate girders the horizontal stiffeners are purely stiffeners, and the highest efficiency should be effected.

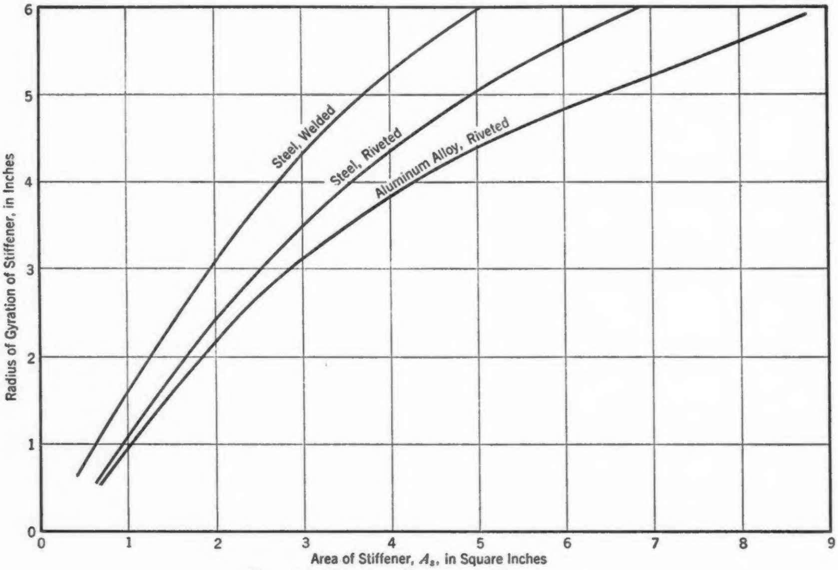


Fig. 1.—Highest Stiffener Efficiency

Longitudinal Stiffeners in Plates of Columns

The proper spacing and proportioning of longitudinal stiffeners increase the allowable plate slenderness to multiples of the unreinforced column plate, depending on the number of stiffeners used. This warrants the use of thinner plates and allows a greater part of the material to be placed on the periphery of the column where it is most effective. In cellular structures, such as towers of suspension bridges, or large posts of other bridges, the use of longitudinal stiffeners permits larger cells. This simplifies the fabrication and erection of the structures.

TABLE 5.—Terms for Use in Eqs. 19 and 20 When Compression Is Uniform and Stiffeners Are Equidistant

Description	Number of Stiffeners, N		
	1	2	3
Equivalent coefficient, k' (for each case $k_0 = 4$)	16	36	64
$\Sigma \delta_s \sin^2 \pi c_s$ (see Eq. 20)	δ	1.5δ	2.0δ

Size and Spacing of Longitudinal Stiffeners.—For uniform compression and equal and equidistant stiffeners, the terms in Eqs. 19 and 20 are as shown in

Table 5. With these established values, Eq. 19 can be written,

$$\frac{r}{t} = \left\{\frac{\beta^2[2 + 8N + 4N^2 + \delta_s(28 - 32N + 36N^2)] - \beta^4 - 1}{12[1 - (1/m)^2](N + 1)\delta_s}\right\}^{0.5} \cdot (21)$$

for values of

$$\beta \gtreqless [1 + 4N + 2N^2 + \delta(14 - 16N + 18N^2)]^{0.5} \ldots\ldots\ldots (22)$$

In Eq. 21 the term $12\left[1 - \left(\frac{1}{m}\right)^2\right]$ is equal to 10.92 for steel and 10.69 for aluminum, based on a Poisson's ratio of 0.3 and 0.33, respectively. Furthermore, N is the number of stiffeners, δ_s is the ratio of the area of one stiffener to the area of plate, and β the ratio of plate length to maximum allowable plate width d'. For the various materials, the values of d' are as follows:

Material	Maximum allowable plate width, d'
Carbon steel	$50(N + 1)t$
Silicon steel	$40(N + 1)t$
Aluminum alloy	$30(N + 1)t$

The values of δ_s and r in Eq. 19 depend on the size and shape of the stiffeners; therefore, an explicit solution is not possible. The ratio $\frac{r}{t}$ is determined by trial computations using the highest stiffener efficiency, as given in Fig. 1.

For given values of β, t, and N, a value of r is assumed and the corresponding stiffener area is taken from Fig. 1. The value of δ_s is then the area of the stiffener divided by the area of the plate, $d' \times t$. Introducing this value of δ_s into Eq. 21, a value of r will be obtained. The computation should be repeated until the computed value of r agrees with the assumed value. Table 6, which gives the required radius of gyration for various plate thicknesses and values of β, was obtained in this manner and is presented to facilitate computations.

Eq. 20, which expresses values for β for maxima values of $\frac{r}{t}$, indicates that for these limiting ratios no transverse stiffeners will be useful. If the length l is such that β will satisfy Eq. 20, the stiffened plate will buckle in an unrestrained half wave, and a transverse stiffener at these locations will not help to increase the critical buckling stress.

It should be noted that, in the derivation of Eq. 20, the value of δ_s was assumed to be independent of β. It was found that the theoretically correct values of β for maxima values of $\frac{r}{t}$ deviate but slightly from those given by Eq. 20, which justifies the use of this equation in its simplified form. Eq. 20 can be transformed to express minima values of $\frac{r}{t}$ for which no transverse stiffeners are required. The transformed equation reads—

TABLE 6.—RADIUS OF GYRATION REQUIRED PER STIFFENER IN INCHES FOR COLUMN PLATES OF VARIOUS THICKNESSES t AND RATIOS β AND REINFORCED BY ONE, TWO, OR THREE LONGITUDINAL STIFFENERS

Ratio $\beta = l/d$	THICKNESS, t, OF COLUMN PLATE, IN INCHES											
	3/8			1/2			5/8			3/4		
	Number of Stiffeners, N											
	1	2	3	1	2	3	1	2	3	1	2	3
	(*a*) CARBON STEEL; RIVETED											
1.0	1.12	1.60	2.01	1.67	2.34	2.91	2.25	3.12	3.82	2.85	3.90	4.74
1.5	1.49	2.15	2.68	2.19	3.09	3.82	2.92	4.06	4.94	3.66	4.98	5.97
2.0	1.78	2.61	3.28	2.58	3.71	4.59	3.40	4.80	5.83	4.24	5.82	7.09
2.5	1.97	3.01	3.81	2.83	4.25	5.25	3.71	5.40	6.64	4.59	6.56	8.14
3.0	2.05	3.35	4.28	2.91	4.67	5.83	3.79	5.91	7.42	4.67	7.20	9.11
	(*b*) SILICON STEEL; RIVETED											
1.0	1.04	1.50	1.88	1.55	2.19	2.73	2.10	2.92	3.60	2.66	3.66	4.47
1.5	1.41	2.01	2.53	2.05	2.90	3.61	2.73	3.82	4.68	3.44	4.72	5.68
2.0	1.68	2.46	3.10	2.42	3.51	4.38	3.20	4.55	5.54	4.00	5.53	6.74
2.5	1.86	2.85	3.62	2.67	4.02	5.02	3.51	5.16	6.33	4.34	6.23	7.75
3.0	1.95	3.18	4.10	2.76	4.41	5.59	3.60	5.65	7.09	4.44	6.86	8.68
	(*c*) ALUMINUM ALLOY; RIVETED											
1.0	0.94	1.35	1.70	1.40	1.98	2.47	1.88	2.63	3.23	2.38	3.27	3.98
1.5	1.27	1.83	2.32	1.85	2.63	3.27	2.46	3.43	4.19	3.08	4.20	5.05
2.0	1.53	2.25	2.85	2.21	3.18	3.95	2.89	4.08	4.98	3.59	4.93	6.02
2.5	1.72	2.62	3.33	2.45	3.65	4.56	3.19	4.64	5.72	3.93	5.57	6.94
3.0	1.82	2.94	3.78	2.57	4.06	5.09	3.32	5.11	6.43	4.06	6.17	7.83
	(*d*) CARBON STEEL; WELDED											
1.0	1.25	1.75	2.17	1.82	2.53	3.13	2.43	3.36	4.12	3.08	4.21	5.09
1.5	1.63	2.31	2.90	2.36	3.32	4.10	3.14	4.36	5.29	3.95	5.35	6.48
2.0	1.92	2.79	3.50	2.77	3.98	4.92	3.66	5.15	6.24	4.56	6.25	7.60
2.5	2.11	3.21	4.05	3.03	4.53	5.60	3.98	5.79	7.10	4.91	7.02	8.69
3.0	2.17	3.55	4.55	3.08	4.98	6.21	4.03	6.32	7.90	4.97	7.69	9.69
	(*e*) SILICON STEEL; WELDED											
1.0	1.17	1.64	2.04	1.70	2.37	2.93	2.27	3.15	3.88	2.89	3.96	4.81
1.5	1.53	2.17	2.72	2.22	3.12	3.87	2.94	4.10	5.03	3.71	5.07	6.10
2.0	1.80	2.64	3.31	2.60	3.75	4.66	3.44	4.88	5.94	4.28	5.95	7.20
2.5	2.00	3.04	3.85	2.86	4.29	5.34	3.75	5.48	6.74	4.65	6.68	8.24
3.0	2.06	3.38	4.34	2.93	4.73	5.94	3.83	6.02	7.52	4.67	7.31	9.21

For steel:

$$\frac{r}{t} = (4 + 2\,N^2)\,\frac{d}{d'} \qquad \text{(23a)}$$

and for aluminum alloy:

$$\frac{r}{t} = (2.5 + 3\,N^2)\,\frac{d}{d'} \qquad \text{(23b)}$$

in which d is the actual and d' the basic allowable plate width.

It will be noted that the theoretical plate width is introduced in both Eqs. 21 and 23. This was done to adapt the equations to practical requirements. In the actual design of columns the full allowable plate width can seldom be applied because other limitations have to be considered to effect a balanced structure. Thus it often may be the case that the longitudinal stiffeners are spaced closer than theoretically required. This will increase the buckling stability of the panel above the required limits; but there is no reason why the stiffened plate as a whole should have the same increased buckling strength.

Transverse Stiffeners in Plates of Columns

In the analysis of the behavior of plates reinforced by longitudinal stiffeners, it was assumed that the transverse stiffeners will prevent buckling at their locations. A method had to be found to determine the size of transverse stiffeners to act as nodal lines. A mathematical approach to this problem is rather involved, and it is doubtful that an exact analytical solution is possible. The procedure adopted to approach a solution of this problem follows from the knowledge of the buckling behavior of plates.

An unreinforced plate subject to uniform compressive stresses will buckle into waves; the number of these waves depends on the ratio of length to width of plate. It is evident that if a transverse stiffener is placed at the nodal line of the plate the latter will not increase its buckling strength. However, should transverse stiffeners be placed in such a manner that they will shorten the length of each half wave, the buckling stability will be increased. Therefore, the problem was approached as follows:

A panel of a column plate reinforced by longitudinal stiffeners was virtually transformed into an orthogonally isotropic plate of the same width but of a reduced length. The buckling resistance of a panel of length l and depth d is proportional to its rigidity in longitudinal and transverse directions and, inversely, proportional to the fourth power of the dimensions of l and d. This can be deduced from the deflections of strips of the plate acting at right angles to each other; at their intersection the deflections must be identical. To transform the panel into a virtual plate of equal rigidities in both directions, the length l_s should be reduced by the ratio ρ. It also can be deduced from this that if the actual ratio β of the panel is such that it is equal to ρ the least critical stress will be obtained. In other words, transverse stiffeners placed in accordance with this expression will not increase the buckling stability of the plate.

A number of examples have been computed and it was found that a general approximation for the ratio $\frac{1}{\rho}$ may be expressed as a function of β and N, namely,

$$\frac{1}{\rho} = 0.50\sqrt{\frac{\beta}{N}}\frac{1}{\beta} \qquad (24)$$

This relation makes it possible to establish the virtual ratio $\beta' = \frac{\beta}{\rho}$. The panel can thus be transformed into an orthogonally isotropic plate with a length-to-width ratio of β'.

It is reasoned that, if it is reinforced by transverse stiffeners, this plate should develop the same critical stress as the actual plate reinforced by longitudinal stiffeners. For the actual panel the general expression for the critical stress, Eq. 1, is

$$\sigma_{cr} = k_1 \frac{\pi^2 E \zeta}{12\left[1 - \left(\frac{1}{m}\right)^2\right]} \left(\frac{t}{d}\right)^2 \ldots\ldots\ldots\ldots\ldots\ldots (25a)$$

and, for the virtual plate between transverse stiffeners,

$$\sigma_{cr} = k_2 \frac{\pi^2 E \zeta}{12\left[1 - \left(\frac{1}{m}\right)^2\right]} \left(\frac{t_v}{d}\right)^2 \ldots\ldots\ldots\ldots\ldots\ldots (25b)$$

in which t_v is the virtual thickness. The value k_1 is 16, 36, and 64 for $N = 1$, 2, and 3, respectively. The value k_2 can be established for various values of β' or β.

By equating the two values of the critical stress, an expression for t_v can be established. From $k_1 = 4\,(N + 1)^2$, and $k_2 = \left(0.50\sqrt{\frac{\beta}{N}} + 2.0\sqrt{\frac{N}{\beta}}\right)^2$,

$$t_v = \frac{2\,(N + 1)}{0.50\sqrt{\frac{\beta}{N}} + 2.0\sqrt{\frac{N}{\beta}}}\, t \ldots\ldots\ldots\ldots\ldots\ldots (26)$$

A close approximation for t_v is given by

$$t_v^3 = 4\,\beta\, N\, t^3 \ldots\ldots\ldots\ldots\ldots\ldots (27)$$

The ratio of the rigidity of a transverse stiffener to the virtual plate is

$$\gamma_v = \frac{12\left[1 - \left(\frac{1}{m}\right)^2\right]}{t_v^3\, d} I_t \ldots\ldots\ldots\ldots\ldots\ldots (28)$$

or, substituting Eq. 27 in Eq. 28, the ratio becomes:

$$\gamma_v = \frac{12\left[1 - \left(\frac{1}{m}\right)^2\right]}{4\,\beta\, d N\, t^3} I_t \ldots\ldots\ldots\ldots\ldots\ldots (29)$$

An approximate critical value of this ratio can theoretically be established for any number of transverse stiffeners. In practical design the value of β' is between 0.3 and 0.7. For this range an approximate expression for γ_v is

$$\gamma_v = \frac{0.018\left\{12\left[1 - \left(\frac{1}{m}\right)^2\right]\right\}}{(\beta')^3} \sqrt{N'} \ldots\ldots\ldots\ldots\ldots\ldots (30)$$

in which N' = number of transverse stiffeners and $\beta' = \frac{\beta}{\rho}$.

Each stiffener should be so proportioned that the value of γ_v will not be less than given by Eq. 30. A combination of Eqs. 29 and 30 will give an approximate value for the moment of inertia I_t required for one transverse stiffener:

$$I_t = 0.58\sqrt{\frac{N^5}{\beta}}\, t^3\, d\, \sqrt{N'} = T\, t^3\, d\, \sqrt{N'} \quad \text{.............. (31)}$$

for values of $\beta \gtreqless 3.2\, N$. The factor T for various values of β and N' is given in Table 7.

TABLE 7.—Values of Coefficient T, in Eq. 31

Number of stiffeners, N	Coefficient T for the Following Values of $\beta = l/d$:												
	0.8	1.0	1.2	1.4	1.6	1.8	2.0	2.5	3.0	3.5	4.0	4.5	5.0
1	0.648	0.580	0.529	0.490	0.459	0.432	0.410	0.367	0.335	....	....	....	....
2	3.67	3.34	3.00	2.77	2.60	2.44	2.32	2.08	1.90	1.75	1.64	1.54	1.47
3	10.10	9.04	8.26	7.64	7.16	6.73	6.39	5.72	5.22	4.83	4.52	4.25	4.04

Eq. 31 and Table 7 apply to steel as well as to aluminum. The table indicates smaller transverse stiffeners with an increasing ratio β. This is in accord with the theory. Denoting the unit rigidity of a plate in a longitudinal (R_a) and a transverse (R_b) direction, respectively, by:

$$R_a = \frac{E\, t^3}{12\left[1 - \left(\frac{1}{m}\right)^2\right]} + \frac{E\, I_s}{b_w\, t} \quad \text{.............. (32a)}$$

and

$$R_b = \frac{E\, t^3}{12\left[1 - \left(\frac{1}{m}\right)^2\right]} \quad \text{.................. (32b)}$$

no transverse stiffeners will be required if $\beta = \sqrt[4]{\frac{R_a}{R_b}}$. With a decreasing ratio, β, the required size of the transverse stiffeners increases. This is also in accord with the theory because it forces the plate into shorter half waves.

Vertical Stiffeners of Webs of Plate Girders

Vertical stiffeners or intermediate stiffeners of plate girders prevent the web from buckling below the desired critical stress caused by shear. Shear is fundamentally a tension stress and can be compared to pure tension with the understanding that the yield strength for shear is below the yield strength in tension. For steels this reduction is usually taken to be 36%, resulting in a yield strength of 23,000 lb per sq in. for carbon steel and 28,800 lb per sq in. for silicon steel; the corresponding value for the aluminum alloy is 30,000 lb per sq in. The proportional limit is about 80% of the yield strength; the same reduction may be taken for shear. The values for the proportional limits in shear are: For carbon steel 18,400, for silicon steel 23,000, and for the aluminum alloy 24,000, lb per sq in.

The moduli of elasticity in shear below the values representing the proportional limit are constant. Above the proportional limit the moduli will decrease, theoretically, approaching zero at the yield point. The transition curve between the proportional limit and the yield point was assumed to be an ellipse as the best approximation derived from the shear stress-strain curves of the materials. With these assumptions the value of ζ can be computed for various critical stresses. The results are compiled in Table 8. The allowable

TABLE 8.—VALUES OF MODULUS FACTOR ζ FOR VARIOUS CRITICAL STRESSES

Meta	Modulus Factor ζ for the Following Values of Critical Shearing Stress, τ_{cr}, in Kips per Square Inch:											
	18	19	20	21	22	23	24	25	26	27	28	29
Carbon steel	1.000	0.994	0.953	0.867	0.706	....	....	....	....	....	....	....
Silicon steel	1.000	1.000	1.000	1.000	1.000	1.000	0.989	0.954	0.890	0.788	0.609	....
Aluminum alloy	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.990	0.957	0.898	0.804	0.648

unit shear for carbon steel is 11,000, for silicon steel 14,000, and for the aluminum alloy, 12,500, lb per sq in. With a factor of safety of 1.5, the critical shearing stresses become 16,500, 21,000, and 18,750, lb per sq in., respectively.

Spacing of Vertical Stiffeners.—It will be noted that the proportional limit for each material is greater than the foregoing critical buckling stress. Therefore, the value of ζ in Eq. 1 can be taken as unity; in other words, the modulus of elasticity is constant for shear, 29,000,000 lb per sq in. for steel, and 10,300,000 lb per sq in. for aluminum.

In Eq. 1, let: d = width of web between the toes of the flange angles, l_s = the spacing of intermediate stiffeners, $\beta = \frac{l_s}{d}$, τ = unit shear in web, and τ_{cr} = critical shearing stress = $1.5\,\tau$. Then, solving for l_s,

$$l_s = \frac{\beta\, t \left\{ k \frac{\pi^2 E}{12\,[1 - (1/m)^2]} \right\}^{0.5}}{(1.5\,\tau)^{0.5}} \quad \text{..............(33)}$$

Table 1 gives values of k for various ratios β. Introduced in Eq. 33, the expression for l_s can be written

$$l_s = \text{coefficient} \times \frac{t}{\sqrt{\tau}} \quad \text{....................(34)}$$

The values of the coefficient for various ratios β are:

β	For steels	For aluminum
0.2	10,910	6,570
0.4	10,370	6,240
0.6	10,870	6,540
0.8	11,700	7,050
1.0	12,750	7,680

It will be noted that the coefficient changes little for various values of β; and, therefore, it is justifiable to eliminate β for practical requirements. With this in view, it is proposed to use, for the intermediate stiffener spacing, the following simple expressions—

For steel:

$$l_s = 10{,}500 \frac{t}{\sqrt{\tau}} \qquad \text{(35a)}$$

and for the aluminum alloy:

$$l_s = 6{,}500 \frac{t}{\sqrt{\tau}} \qquad \text{(35b)}$$

Not all girder webs require intermediate vertical stiffeners, which can be demonstrated by applying the general stability expression, Eq. 1. For long, simply supported plates the value of k is 5.35. Taking the critical shearing stress as one and one half times the actual shear, the modulus factor ζ is unity, and the clear depth, d (for which no intermediate stiffeners are required) becomes

For steel:

$$d = \frac{9{,}600\, t}{\sqrt{\tau}} \qquad \text{(36a)}$$

and for aluminum:

$$d = \frac{5{,}800\, t}{\sqrt{\tau}} \qquad \text{(36b)}$$

In these expressions the factors 9,600 and 5,800 have been "rounded out" from 9,669 and 5,823, respectively. Table 9 gives a few values of the maximum

TABLE 9.—MAXIMUM ALLOWABLE CLEAR DEPTH, d, IN INCHES, OF WEBS REQUIRING NO INTERMEDIATE STIFFENERS

Material	Thickness, t, in inches	Depths, in Inches, for the Following Values of Web Stress, τ, in Kips per Square Inch:							
		1	2	3	4	5	6	8	10
Steel	3/8	113.8	80.5	65.7	56.9	50.9	46.5	40.3	36.0
Aluminum	3/8	68.8	48.6	39.7	34.4	30.8	28.1	24.3	20.7
Steel	1/2	151.8	107.3	87.6	75.9	67.9	62.0	53.7	48.0
Aluminum	1/2	91.7	64.8	52.9	45.9	41.0	37.4	32.4	27.7
Steel	5/8	189.7	134.2	109.5	94.9	84.8	77.5	67.1	60.3
Aluminum	5/8	114.6	81.1	66.2	57.3	51.3	46.8	40.5	34.6

allowable clear depth of webs for which no intermediate stiffeners are required.

Size of Vertical Stiffeners.—To determine the required size of vertical stiffeners it should be kept in mind that they may be called upon to transmit load concentrations, or shear, or both, and act as stiffeners as well. In the first case, depending on the load concentration, a symmetrical stiffener may be of advantage, whereas in the latter case a stiffener on one side of the web only will show greater efficiency. It is prudent, therefore, to design the vertical stiffeners as columns and as stiffeners. The column should be assumed to consist of the stiffener and a strip of web equal to twenty-five times its thickness; the

column length may be taken at three quarters of the actual length. To assure stability under all conditions, the load taken for the assumed column should consist of the load concentration and part of the shear, depending on the spacing of the stiffener. The following load is proposed:

$$P = V\beta + P_1 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (37)$$

but not more than $(V + P_1)$. In Eq. 37: V = average of values of shear at left and right side of stiffener, in lb; P_1 = local load concentration, in lb; and β = ratio of stiffener spacing to total height of web (not to exceed 1.0).

A mathematical procedure to evaluate the required size of a stiffener is difficult and tedious for the most simple assumptions and becomes practically impossible for actual conditions. The rigorous investigations made by E. Chwalla and A. Novak (44) proved that a previous, simple proposal by Professor Chwalla (36) gives satisfactory values for design purposes. This proposal was adopted. Translated into the nomenclature of this paper, it reads:

$$I_s = \frac{(0.1 + 0.02\,N)\,t^3\,d}{\beta_1^4}\,(\beta_1^2 + 0.625) \quad \ldots\ldots\ldots\ldots (38)$$

but not to exceed $\frac{0.2\,t^3\,d}{\beta_1^4}\,(\beta_1^2 + 0.625)$. In Eq. 38: β_1 = ratio of the theoretically required stiffener spacing l_s to total depth of web (in cases where l_s changes, the average value of the spacing should be used to compute β_1. For values of β_1 less than 0.4, use 0.4 for β_1); and I_s = moment of inertia of stiffener, in inches^4.

The assumptions made in the derivation of the foregoing empirical formula (Eq. 38) for the required moment of inertia limit the ratio β between 0.4 and 2.5. Values of β outside these limits would lead to results which are invalid. The upper limit is of no practical importance since the ratio β is always below 2.5 in plate-girder design. Values of β of less than 0.4 may occur, however, and it is necessary to make provision for these cases. For these values the theory of orthogonally anisotropic plates may be applied (15) (19) (28) (29). A great number of examples of webs were analyzed with an intermediate stiffener spacing giving ratios β of 0.4 and less. It was found that the proposed formula for the intermediate stiffeners gives an ample section if a value of β_1 of 0.4 is used for all cases in which β is less than 0.4. The required size of intermediate stiffeners thus determined is also in agreement with tests made by E. Seydel (15) (19) (28) (29). For equal stiffeners on both sides of the web, the moment of inertia should be taken about the center line of the web. For stiffeners on one side only, the moment of inertia should be taken about the face of the web in contact with the stiffener.

It will be noted that in Eq. 38 the theoretically required spacing l_s is used in the ratio β_1. The reasons for this are simply to accommodate the formula to practical design considerations. Stress conditions require a certain stiffener spacing which may not be fully realized in the actual design. The spacing of intermediate stiffeners may be made considerably smaller than theoretically required to aline them to stringers in the case of floor beams or between floor beam connections in the case of plate girders.

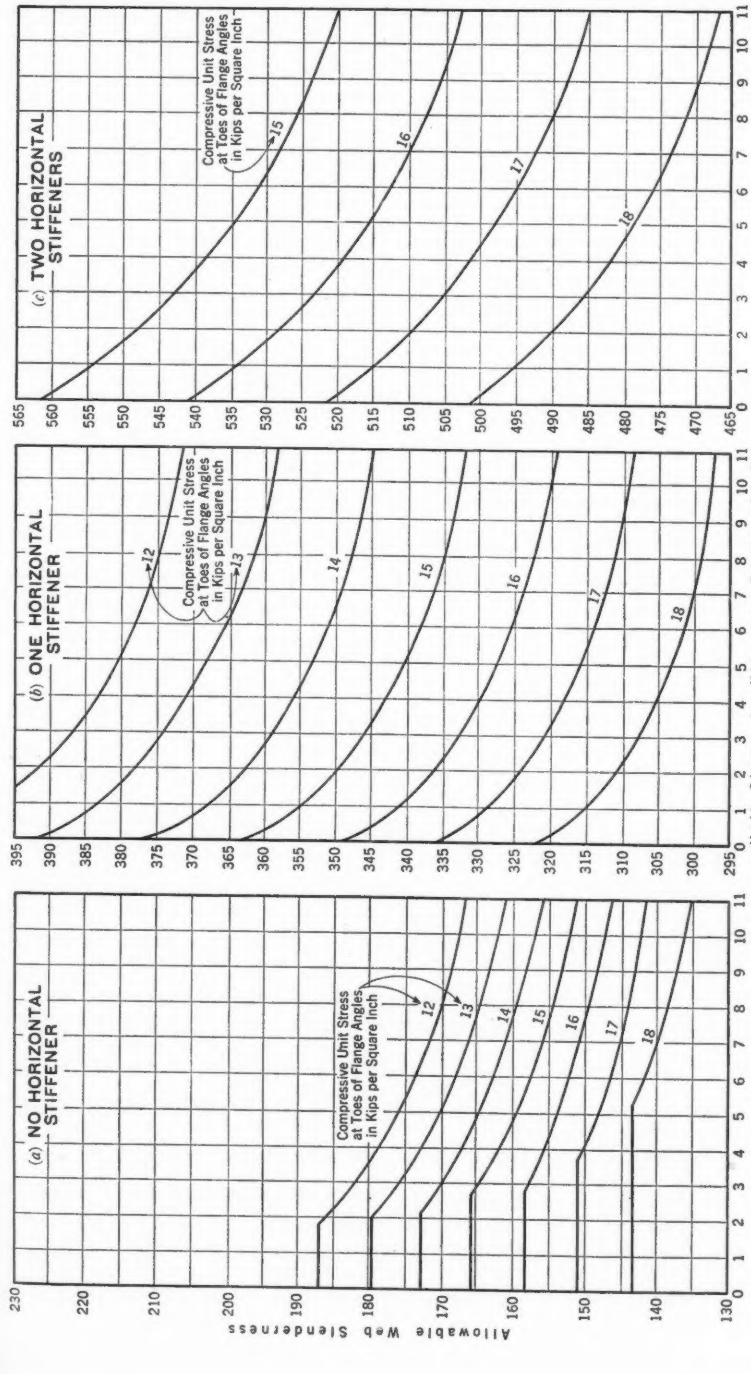


FIG. 2.—ALLOWABLE SLENDERNESS RATIOS $\frac{d}{t}$ FOR CARBON STEEL

The smaller spacing increases the buckling stability of the web panel above the required limit. Since the formula for the size of the stiffener is balanced with the buckling strength of the web, an equally higher strength in the stiffened web would be attained by using the actual spacing. This is not necessary.

Horizontal Stiffeners of Webs of Plate Girders

The general buckling behavior of the web of plate girders subject to load was discussed briefly under the heading of "Design of Webs for Plate Girders," Chapter 3. Buckling of the web is caused by compression or shear, or both. As it is desirable to have a buckling safety factor of 1.5 for compression or shear and one of 1.4 for the combined action of the two, a more detailed study of the behavior of the web becomes necessary.

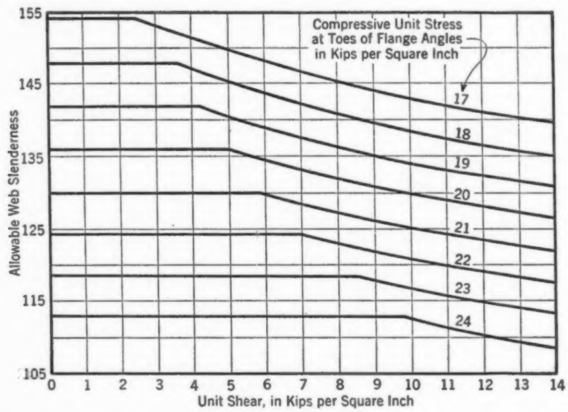


FIG. 3.—ALLOWABLE SLENDERNESS RATIOS $\frac{d}{t}$ FOR SILICON STEEL; NO HORIZONTAL STIFFENERS

To clarify the interdependence of stresses on the buckling stability of the web, the general stability expression, Eq. 1, will first be applied to the unstiffened web of a plate girder.

Carbon Steel with No Horizontal Stiffeners ($N = 0$).—The plate is in pure bending, and $\alpha = 2$ in Eq. 2. Three examples will serve to demonstrate this case.

Example 1.—Maximum compression at toe of flange angles $\sigma = 15{,}000$ lb per sq in. and shear $\tau = 0$. The critical stress is $\sigma_{cr} = 1.5 \times 15{,}000 = 22{,}500$ lb per sq in. By Eq. 1, $22{,}500 = 26{,}210{,}000\, \zeta\, k \left(\frac{t}{d}\right)^2$. For $\alpha = 2$, Eq. 3 gives $k = 24$. For a critical stress of 22,500, Table 3 gives a value of 0.979 for ζ. Introducing these values in Eq. 1, the web slenderness $\frac{d}{t} = 165$..

Example 2.—Maximum compression at toe of flange angles $\sigma = 18{,}000$ lb per sq in. and shear $\tau = 0$. The critical stress now increases to 27,000 lb as in Example 1, the value of coefficient k is again 24, but the value for ζ drops to 0.878. The web slenderness becomes, similarly, $\frac{d}{t} = 143$. To study the effect

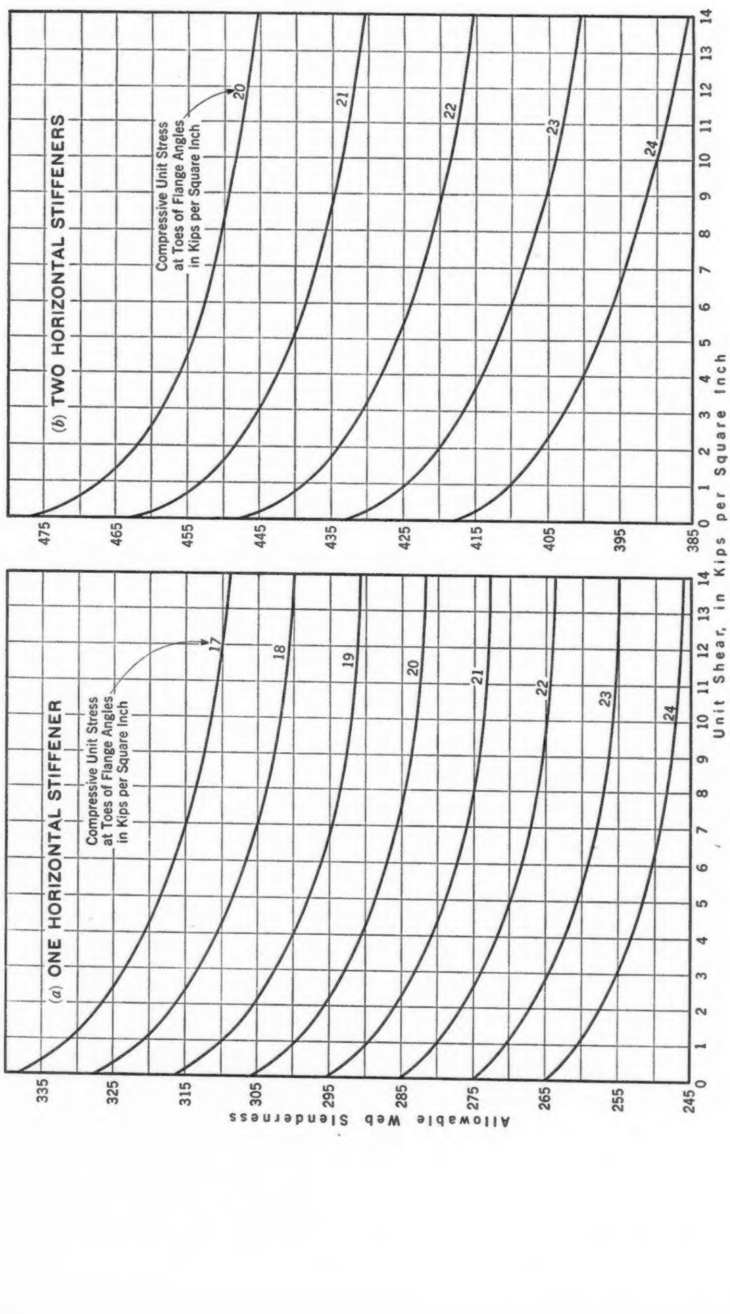


FIG. 4.—ALLOWABLE SLENDERNESS RATIOS $\frac{d}{t}$ FOR SILICON STEEL

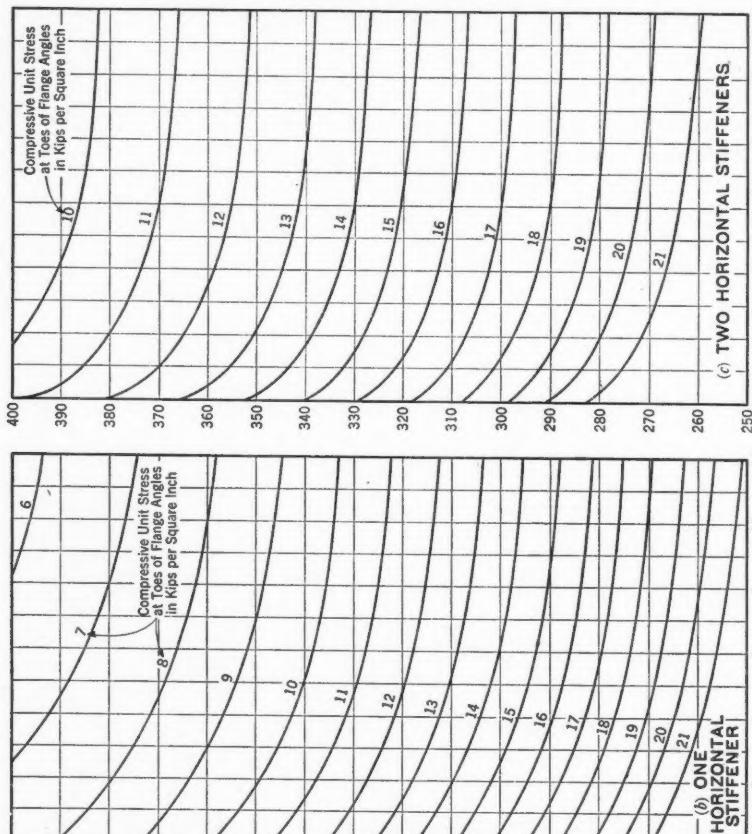


FIG. 5.—ALLOWABLE SLENDERNESS RATIOS $\frac{d}{t}$ FOR ALUMINUM ALLOYS

of the combined action of compression and shear, the following example is given:

Example 3.—Maximum compression $\sigma = 15{,}000$ lb per sq in., and average $\tau = 5{,}000$ lb per sq in. The solution must be found by trial. Assume the factor of safety for compression to be 1.59; σ_{cr} becomes $1.59 \times 15{,}000 = 23{,}850$ lb. The unreduced coefficient k for pure bending is 24, and the value of ζ for the critical stress of 23,850 lb from Table 3 is $\zeta = 0.959$ (see Eq. 5). Eq. 1 reads, $23{,}850 = 26{,}210{,}000 \times 0.959 \times 24 \left(\frac{t}{d}\right)^2$, or $d = 159\,t$.

For the spacing of vertical stiffeners it has been proposed to use $l_s = 10{,}500 \times \frac{t}{\sqrt{\tau}} = 148.5\,t$; the ratio β becomes $\frac{148.5}{159} = 0.93$; the coefficient k for shear for this value of β is (see Table 1) $k_s = 10.1$; the critical shearing stress (Eq. 1) is $\tau_{cr} = 26{,}210{,}000\,\zeta\,10.3 \left(\frac{t}{d}\right)^2$; and, the value of d being $159\,t$, the critical shear $\tau_{cr} = 10{,}490\,\zeta$. Reference to Table 8 shows that ζ is unity, and $\tau_{cr} = 10{,}490$ lb per sq in.

The ratio of the actual shear τ to its critical value is $5{,}000 : 10{,}490 = 0.477$. Table 2 shows that for this ratio the reduced coefficient k_d is 85.4%. The shear thus reduces the coefficient k for axial stresses from 24 to 85.4% of this value, or 20.5. The critical axial stress, therefore, is reduced to $\sigma_{cr} = 26{,}210{,}000\,\zeta \times 20.5 \left(\frac{1}{159}\right)^2 = 21{,}250\,\zeta$. A value of ζ must be assumed by trial until σ_{cr} and $21{,}250\,\zeta$ balance in accordance with Table 3. It will be seen that ζ is practically unity and σ_{cr} becomes 21,100 lb. The factor of safety is 1.41, or practically 1.4, as desired. The ratio of depth to thickness, or the web slenderness, can be established similarly for any desired combination of axial stress and shear. For $\sigma = 15{,}000$ lb and $\tau = 11{,}000$ lb, $\frac{d}{t}$ will reduce to 150; and, for maximum compression and maximum shear, $\sigma = 18{,}000$ lb and $\tau = 11{,}000$ lb, it will reduce further to 135. The web slenderness thus depends on the magnitude of the compressive stress at the toe of the flange angles, on the magnitude of unit shear, and on the combination of the two stresses.

Spacing of Horizontal Stiffeners.—The proper placing of horizontal stiffeners increases the buckling strength of the web; for the same critical stresses, the presence of horizontal stiffeners permits greater web slenderness. The spacing of these stiffeners demands equal buckling strength for the individual panels formed by the horizontal and vertical stiffeners. The analysis to determine allowable web slenderness and spacing of stiffeners follows the examples shown for simple plate girders.

An investigation is shown here for the aluminum alloy[3] with an axial stress of 17,000 lb per sq in. at the toe of the flange angles in combination with the maximum allowable shear of 12,500 lb. Such a loading may occur at a support of a continuous plate girder. Two horizontal stiffeners are assumed. It is desired to find the allowable web slenderness and the spacing of the horizontal stiffeners. Assuming the factor of safety for compression as 1.55, the

critical stress at the compression toes of the flange angles becomes 1.55 × 17,000 = 26,350 lb per sq in. Let it be further assumed that—if $c_1\,d$ and $c_2\,d$ are the distances, respectively, of the first and second stiffeners to the toe of the compression flange angle—$c_1 = 0.129$ and $c_2 = 0.285$. With these assumptions the buckling stability of the individual panels can be determined. The assumptions must be changed until the computations show a balanced buckling strength.

The spacing of vertical stiffeners is derived from $l_s = 6{,}500 \frac{t}{\sqrt{\tau}} = 58.138\,t$ (approximately 58 t). The assumed horizontal stiffener spacing gives the compressive stresses at their location. They are as follows:

Location	Compressive stress, in pounds per square inch
At the toe of flange angles	= 17,000
At first stiffener	= 12,610
At second stiffener	= 7,310

All values for computing α in Eq. 2 are thus known. For the three panels: $\alpha_1 = 0.258$; $\alpha_2 = 0.421$; and $\alpha_3 = 3.326$. Introducing these values in Eq. 3: $k_1 = 4.217$; $k_2 = 4.605$; and $k_3 = 73.959$. The coefficients k are the unreduced values for compression. Applying Eq. 1 to the first panel,

$$26{,}350 = 4.217 \times 9{,}507{,}000\,\zeta \times \left(\frac{t}{c_1\,d}\right)^2 \ldots\ldots\ldots\ldots\ldots (39)$$

The value of ζ from Table 3 is 0.965 for the critical stress of 26,350 lb; from Eq. 39 the depth becomes $d = 297.05\,t$. The depths of the individual panels are $d_1 = 38.32\ t$, $d_2 = 46.34\ t$, and $d_3 = 212.39\ t$. Since the length of each panel is 58.138 t, the length-to-depth ratio becomes $\beta_1 = 1.52$, $\beta_2 = 1.26$, and $\beta_3 = 0.273$. With these values of β, Table 1 yields: $k_1 = 7.08$, $k_2 = 7.88$, and $k_3 = 85.87$. Introduced in Eq. 1, the critical shear for each panel is $\tau_{1cr} = 45{,}840\,\zeta_1$, $\tau_{2cr} = 34{,}890\,\zeta_2$, and $\tau_{3cr} = 18{,}100\,\zeta_3$.

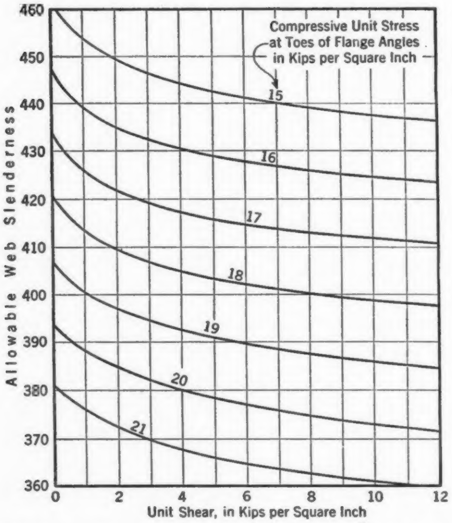


FIG. 6.—ALLOWABLE SLENDERNESS RATIOS $\frac{d}{t}$ FOR ALUMINUM ALLOY, WITH THREE HORIZONTAL STIFFENERS

Table 8 gives the values of ζ by balancing both sides of the foregoing expressions for the critical shearing stress. This balancing is done by trial, as follows:

The second expression reads $\tau_{2cr} = 34{,}890\,\zeta_2$. As a first trial, $\zeta_2 = 0.80$, or $\tau_{2cr} = 34{,}890 \times 0.80 = 27{,}910$. For $\tau_{cr} = 27{,}000$, Table 8 gives a value of $\zeta = 0.898$, and, for $\tau_{cr} = 28{,}000$, $\zeta = 0.804$. The assumed value for ζ_2 of 0.80, giving a critical stress of 27,910 lb, is thus closely correct. The balanced

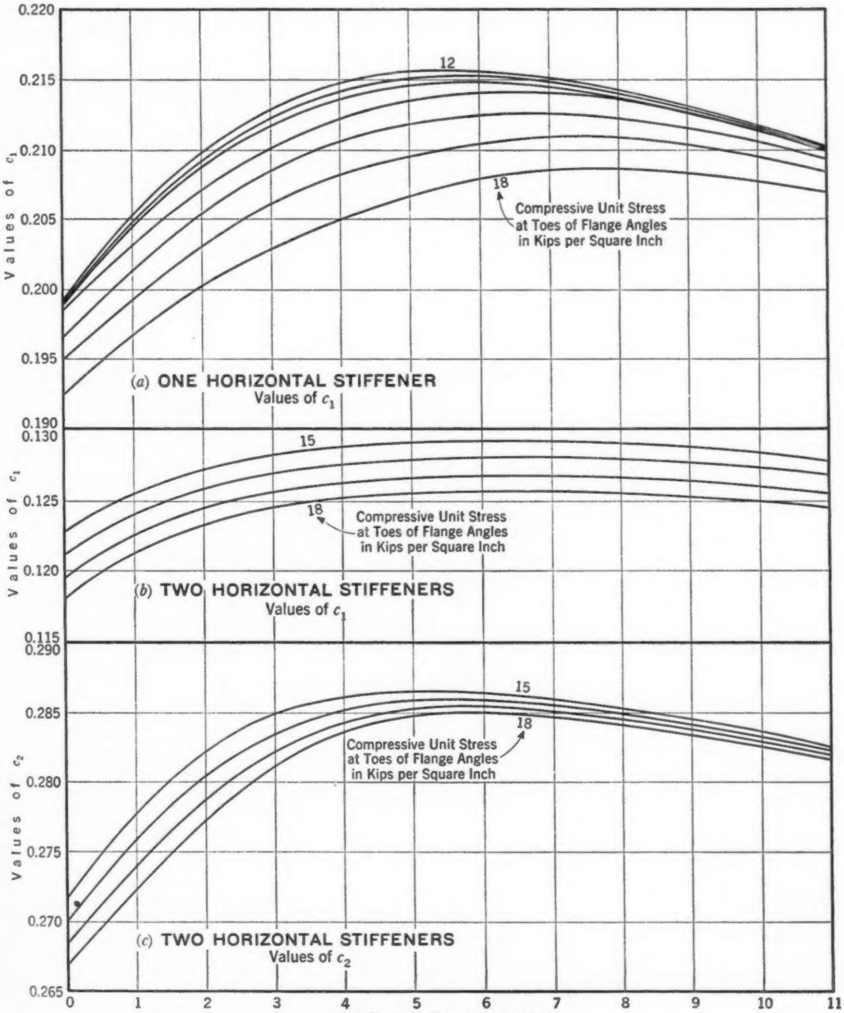


FIG. 7.—SPACING OF HORIZONTAL STIFFENERS; CARBON STEEL

values for the modulus coefficients are: $\zeta_1 = 0.63$, $\zeta_2 = 0.80$, and $\zeta_3 = 1.0$. The critical stresses, therefore, are $\tau_{1cr} = 28{,}880$ lb, $\tau_{2cr} = 27{,}910$ lb, and $\tau_{3cr} = 18{,}100$ lb. The ratio of the actual shear to its critical value becomes 0.43, 0.45, and 0.69, respectively. The reduction in the stability coefficient k

for bending stresses due to the simultaneous action of shear becomes (see Table 2): $k_{1d} = 0.88\,k_1$, $k_{2d} = 0.87\,k_2$, and $k_{3d} = 0.68\,k_3$. Introducing, for k_1, k_2, and k_3, the computed values of 4.217, 4.605, and 73.959, respectively, the

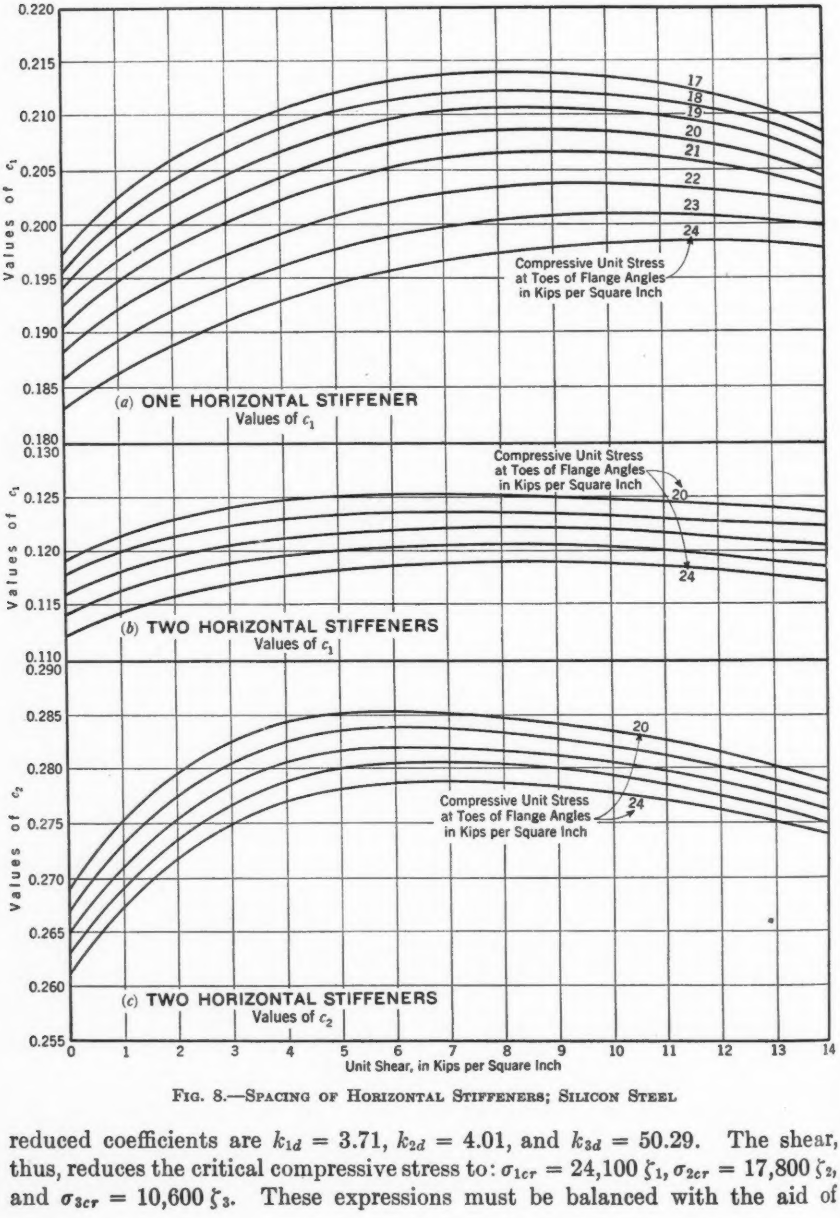


FIG. 8.—SPACING OF HORIZONTAL STIFFENERS; SILICON STEEL

reduced coefficients are $k_{1d} = 3.71$, $k_{2d} = 4.01$, and $k_{3d} = 50.29$. The shear, thus, reduces the critical compressive stress to: $\sigma_{1cr} = 24{,}100\,\zeta_1$, $\sigma_{2cr} = 17{,}800\,\zeta_2$, and $\sigma_{3cr} = 10{,}600\,\zeta_3$. These expressions must be balanced with the aid of

Table 3, similarly to the foregoing balancing of shears: ζ_1 is 0.99, and ζ_2 and ζ_3 are unity. The critical stresses thus become 23,840, 17,800, and 10,600 lb per sq in.; and the corresponding actual stresses are 17,000, 12,610, and 7,310 lb per sq in., as originally assumed. The factors of safety against buckling, therefore, are 1.40, 1.41, and 1.45 for the upper, middle, and lower panels, respectively, which is in close agreement with the desired buckling strength.

The procedure to compute the horizontal stiffener spacing for the webs of plate girders has been shown. It should be realized that a certain degree of familiarity with the buckling theory is necessary and that much labor is required to attain a satisfactory balance of factors of safety for the various panels. For these reasons it was thought desirable to present diagrams that give the required stiffener spacing for various stress conditions. A great number of computations for various stress combinations have been made and the results are shown in Figs. 2 to 9. The allowable web slenderness and the spacing of horizontal stiffeners are shown in the diagrams for a wide variation and combination of stresses. The allowable web slenderness is highest if no shear is acting, and it increases with decreasing compressive unit stress at the toes of the flange angles. The effect of horizontal stiffeners on the allowable web slenderness is revealed clearly by the diagrams.

Example 4.—For example, assuming a compressive stress at the toe of the flange angles of 15,000 lb per sq in. for carbon steel, 20,000 lb per sq in. for silicon steel, and 17,000 lb per sq in. for the aluminum alloy, acting in conjunction with an average shear of 5,000 lb, the allowable web slenderness is as given in Table 10.

TABLE 10.—ALLOWABLE WEB SLENDERNESS, $\frac{d}{t}$, FOR τ = 5,000 LB PER SQ IN.

Material	Assumed stress, σ, in pounds per square inch	Number of Stiffeners, N			
		0	1	2	3
Carbon steel..........	15,000	159	340	534	....
Silicon steel..........	20,000	136	288	450	....
Aluminum alloy.......	17,000	90	191	301	416

Structural engineers are acquainted with the fact that in present plate-girder designs the unit shear is low in most cases, and the thickness of the web is commonly governed by the allowable web slenderness. The proper placing of horizontal stiffeners increases the allowable web slenderness materially and will effect considerable economy.

Example 5.—For example, assume that a four-lane highway bridge has a simple span of 120 ft. Plate girders are selected for the two main girders. The total depth of each girder is 138.5 in., back to back of 8-in. angles, resulting in a net depth of d = 122.5 in. between toes of flange angles. Carbon steel is used for the girders, and it is assumed that the highest compressive unit stress at the toes of the flange angles is 15,000 lb per sq in. and that this stress prevails

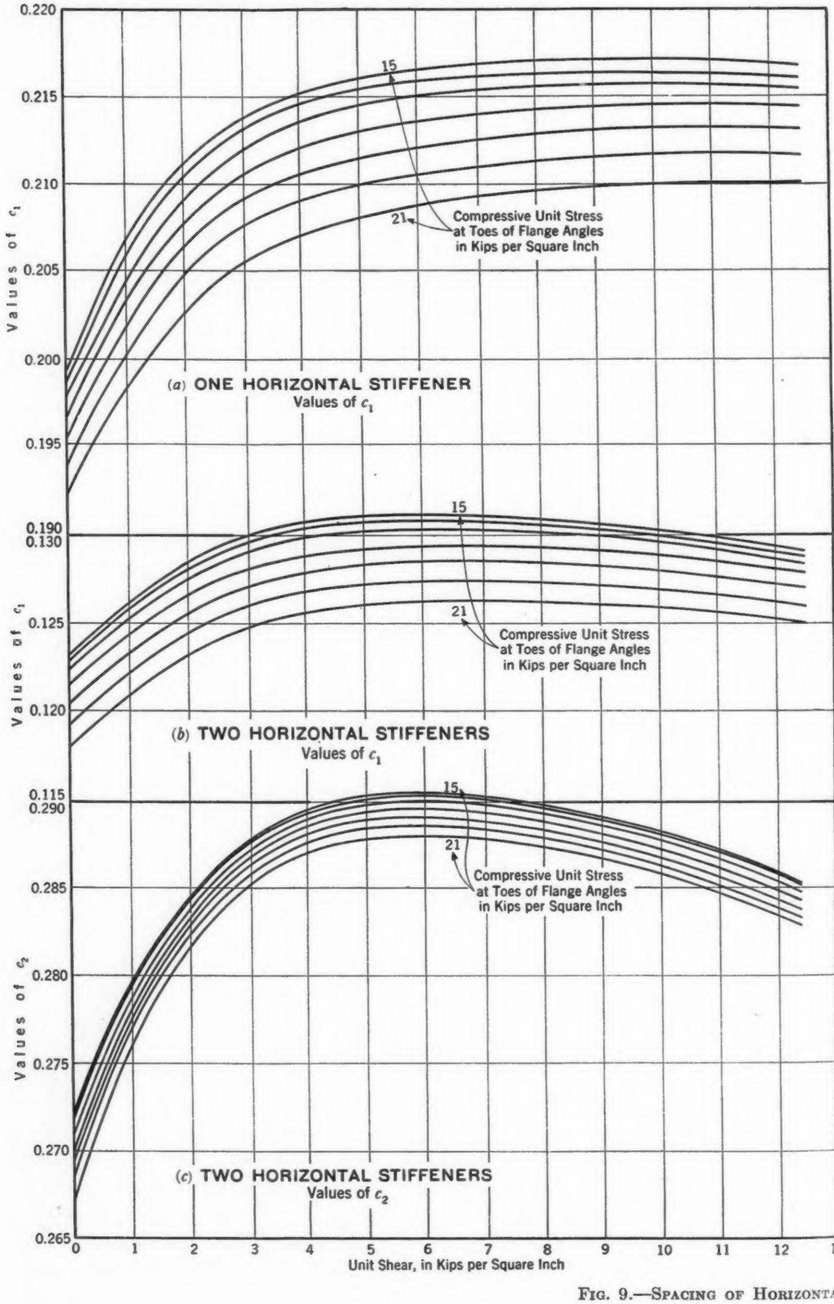
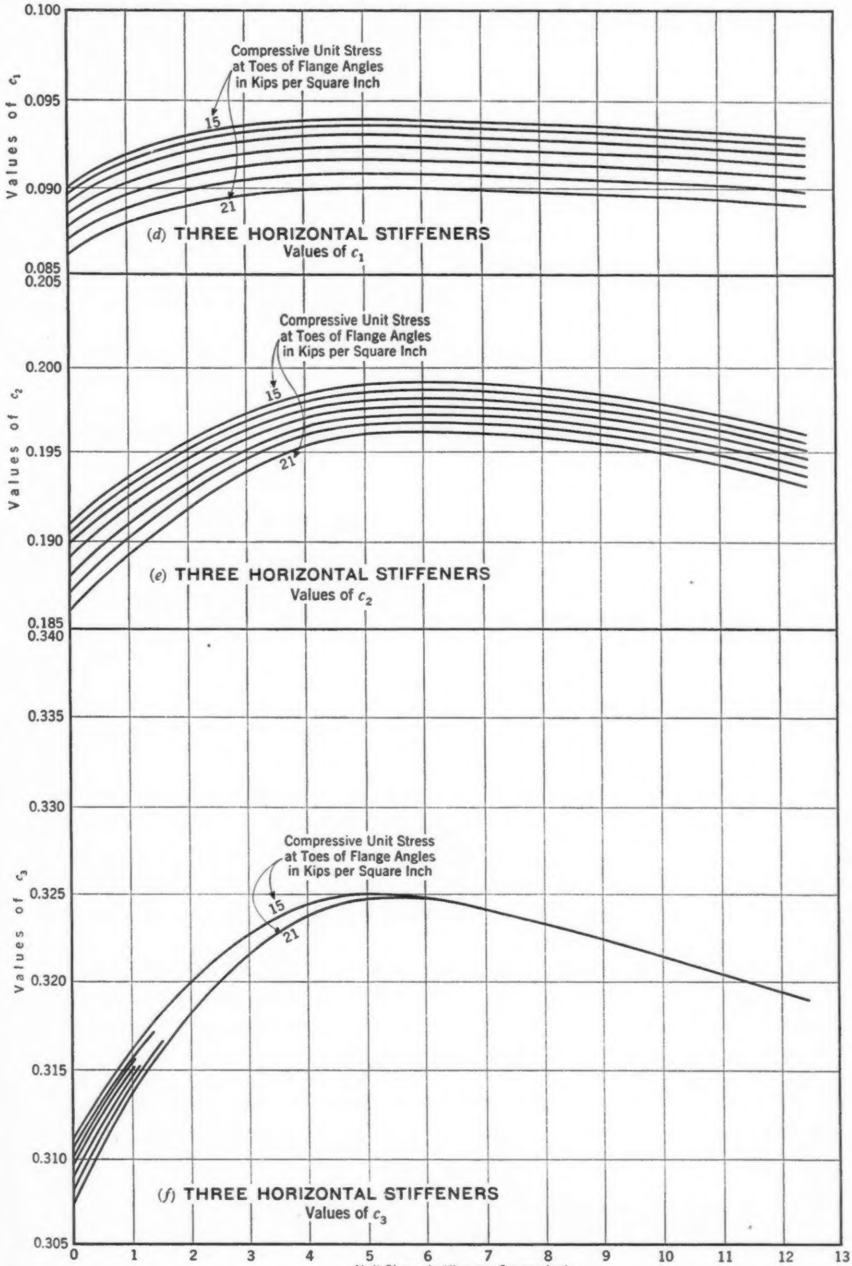


FIG. 9.—SPACING OF HORIZONTAL



STIFFENERS; ALUMINUM ALLOY

not only at midspan, but also near the quarter point where the total shear is taken as 250,000 lb. Therefore, the latter section controls the design.

The plate girder is first proportioned with no horizontal stiffeners. The web thickness is assumed at $t = \frac{3}{4}$ in., resulting (at the controlling section) in a unit shear of 2,400 lb. The allowable web slenderness, from Fig. 2(*a*), for $\sigma = 15{,}000$ and $\tau = 2{,}400$ lb, is 165, which conforms with the assumed value of $\frac{122.5}{0.75} = 163.3$.

With one horizontal stiffener the assumed web thickness is reduced from $\frac{3}{4}$ in. to $\frac{3}{8}$ in., with a corresponding increase in unit shear to 4,800 lb per sq in. The actual web slenderness is $\frac{122.5}{0.375} = 327$. The allowable value, from Fig. 2(*b*), for $\sigma = 15{,}000$ and $\tau = 4{,}800$, is 341; the allowable web slenderness is not even fully utilized, although the web thickness was reduced to one half. The substantial economy effected by horizontal stiffeners is evident. The spacing of the stiffeners is taken from Fig. 7(*a*); it is $0.2134 \times d = 26$ in. from the toes of the upper flange angles.

A study of Figs. 2 to 9 will show the identical behavior of the three materials investigated. The curves for web slenderness show a marked similarity. The required spacing for the horizontal stiffeners is practically the same for carbon steel and the aluminum alloy, whereas, for silicon steel the spacing deviates only slightly. The reason for this behavior is due to identical values of ζ for carbon steel and the aluminum alloy studied. For silicon steel the corresponding values of ζ are somewhat lower because the higher allowable unit stress brings the critical stress nearer to the plastic range.

Size of Horizontal Stiffeners.—To determine the size of horizontal stiffeners of plate girders Eq. 19 must be applied to the problem. The solution follows the pattern of the determination of longitudinal stiffeners in columns, except that the analysis becomes much more involved. The equivalent coefficient k is no longer a multiple of k_0 depending on the number of stiffeners, as in the case of columns. Its magnitude depends on the web slenderness and the critical buckling stress, and these, in turn, are functions of the actual stress conditions. This is illustrated by a simple example of a carbon-steel plate girder with one horizontal stiffener.

Example 6.—It is assumed that the maximum compression at the toes of flange angles is 15,000 lb per sq in. and that no shear is acting. For this condition a factor of safety of 1.5 was taken, which results in a critical stress of 22,500 lb and a value of $\zeta = 0.979$, from Table 3. For the web slenderness Fig. 2(*b*) gives $\frac{d}{t} = 363$. Introducing these values in Eq. 1 and solving for coefficient k,

$$k = \frac{\sigma_{cr}}{26{,}210{,}000\,\zeta}\left(\frac{d}{t}\right)^2 = \frac{22{,}500}{26{,}210{,}000 \times 0.979}(363)^2 = 116$$

Example 7.—The maximum compression at the toe of the flange angles is 18,000 lb per sq in., and the shear is 11,000 lb per sq in. For this stress condi-

tion the minimum factor of safety for combined compression and shear is 1.4; the critical stress becomes 18,000 × 1.4 = 25,200 and $\zeta = 0.930$. The web slenderness from Fig 2(*b*) is 297. These values in Eq. 1 give $k = 91$ for the equivalent coefficient. The evaluation of k can be made for any desired stress condition.

In determining the size of longitudinal stiffeners, the application of Eq. 19, which gives the size of stiffeners, must be made by trial, as shown in the following:

In Example 6, for which $k_0 = 24$ and the equivalent $k_e = 116$, the value of $c_1 = 0.198$, and $\Sigma \sin^2 (\pi\ c_s)$ becomes 0.3395. Introducing these values in Eq. 19:

$$\frac{r}{t} = \left[\frac{(17.3 + 13.1\ \delta)\ \beta^2 - (1 + \beta^4)}{7.415\ \delta_s}\right]^{0.5} \qquad (40)$$

This expression for $\frac{r}{t}$ can be solved for various length-to-height ratios β of the web and for various thicknesses, t, with the aid of Fig. 1.

Example 8.—In the following example $t = \frac{3}{8}$ in., $\frac{d}{t} = 363$, and $\beta = 0.6$. Assume the radius of gyration of the stiffener about the face of the web to be $r = 1.80$ in. For the riveted girder Fig. 1 gives an area of stiffener of 1.55 sq in. The area of the web between the toes of flange angles is 51.05 sq in. The value of δ_s is thus $\frac{1.55}{51.05} = 0.0304$; introduced in Eq. 40, the resulting r becomes 1.80 in., which is in agreement with the assumed value. This procedure to evaluate the size of the horizontal stiffeners can be applied to any plate girder.

The expression for the required size of horizontal stiffeners in implicit form necessitates trial computations which may become involved, and it is desirable to express the required size of stiffeners as a function of readily computed quantities. A great number of trial computations made it possible to express the size of the stiffener as a function of a stiffener coefficient c_r, the web slenderness $\frac{d}{t}$, and the compressive unit stress σ at the toes of the flange angles, as follows:

$$r_1 = c_r \left(\frac{d}{t}\right)^2 \sigma\ 10^{-6} \qquad (41)$$

in which r_1 = required radius of gyration of one stiffener about face of web, in inches.

Values of the stiffener coefficient c_r are given in Table 11 for steel and for the aluminum alloy. With the help of this table, the required size of the horizontal stiffeners can be computed readily.

Example 9.—A 120-ft riveted carbon-steel plate girder is selected to demonstrate the use of this table. The spacing of the vertical stiffeners is assumed to be 50 in. The ratio of length to width of web is then $\beta = 50 : 122.5 = 0.41$. From Table 11(*a*) the value of c_r is found by interpolation to be 0.69 for the

web thickness of $\frac{3}{8}$ in. With $\sigma = 15$ kips per sq in. and $\frac{d}{t} = 327$, the required radius of gyration about the face of the web becomes $r = 0.69 \times (327)^2 \times 15 \times 10^{-6} = 1.11$ in. The smallest angle which permits riveting is $3\frac{1}{2}$

TABLE 11.—VALUES OF STIFFENER COEFFICIENT, c_r

Values of β	Values of c_r for the Following Values of Web Thickness, t, in Inches:											
	5/16	3/8	7/16	1/2	9/16	5/8	5/16	3/8	7/16	1/2	9/16	5/8
	(*a*) Riveted Steel; One Stiffener						(*b*) Welded Steel; One Stiffener					
0.2	0.14	0.25	0.37	0.49	0.61	0.74	0.28	0.38	0.48	0.59	0.70	0.81
0.3	0.35	0.48	0.62	0.77	0.91	1.06	0.45	0.58	0.72	0.85	1.00	1.14
0.4	0.51	0.67	0.84	1.01	1.18	1.36	0.60	0.76	0.93	1.10	1.30	1.46
0.5	0.64	0.83	1.02	1.22	1.43	1.64	0.72	0.92	1.12	1.33	1.54	1.76
0.6	0.75	0.97	1.19	1.42	1.66	1.90	0.83	1.06	1.30	1.54	1.79	2.04
0.7	0.85	1.09	1.34	1.59	1.85	2.12	0.92	1.18	1.44	1.72	1.99	2.28
0.8	0.93	1.19	1.47	1.74	2.03	2.32	1.00	1.28	1.57	1.87	2.18	2.49
0.9	1.00	1.28	1.57	1.87	2.18	2.49	1.07	1.37	1.69	2.01	2.33	2.67
1.0	1.07	1.37	1.67	1.99	2.29	2.64	1.14	1.46	1.79	2.13	2.48	2.84
	(*c*) Riveted Steel; Two Stiffeners						(*d*) Welded Steel; Two Stiffeners					
0.2	0.16	0.22	0.29	0.36	0.43	0.50	0.20	0.27	0.33	0.40	0.48	0.55
0.3	0.23	0.31	0.38	0.46	0.55	0.63	0.27	0.35	0.43	0.52	0.60	0.69
0.4	0.29	0.38	0.47	0.56	0.66	0.76	0.34	0.43	0.52	0.62	0.72	0.82
0.5	0.35	0.45	0.55	0.66	0.77	0.88	0.40	0.50	0.61	0.72	0.83	0.95
0.6	0.41	0.52	0.63	0.75	0.87	0.99	0.46	0.58	0.70	0.82	0.95	1.08
0.7	0.46	0.58	0.70	0.83	0.96	1.09	0.51	0.64	0.77	0.91	1.05	1.19
0.8	0.51	0.64	0.77	0.90	1.04	1.18	0.56	0.70	0.84	0.98	1.13	1.28
0.9	0.55	0.68	0.82	0.97	1.11	1.26	0.61	0.75	0.90	1.05	1.21	1.37
1.0	0.59	0.73	0.87	1.02	1.17	1.33	0.65	0.80	0.95	1.11	1.27	1.44
	(*e*) Riveted Aluminum Alloy; One Stiffener						(*f*) Riveted Aluminum Alloy; Two Stiffeners					
0.2	0.26	0.34	0.43	0.52	0.61	0.71	0.30	0.40	0.51	0.61	0.72	0.84
0.3	0.73	0.95	1.19	1.45	1.72	2.00	0.49	0.65	0.82	0.99	1.16	1.33
0.4	1.09	1.43	1.78	2.15	2.54	2.94	0.65	0.86	1.06	1.27	1.48	1.69
0.5	1.40	1.83	2.26	2.71	3.16	3.63	0.79	1.03	1.27	1.50	1.74	1.97
0.6	1.67	2.16	2.65	3.16	3.66	4.19	0.91	1.18	1.45	1.70	1.96	2.22
0.7	1.90	2.42	2.99	3.53	4.08	4.66	1.03	1.32	1.61	1.89	2.17	2.43
0.8	2.09	2.68	3.29	3.89	4.48	5.11	1.13	1.44	1.75	2.06	2.35	2.63
0.9	2.27	2.92	3.58	4.24	4.86	5.51	1.23	1.56	1.89	2.22	2.51	2.81
1.0	2.45	3.15	3.85	4.55	5.22	5.90	1.31	1.66	2.01	2.36	2.67	2.98
	(*g*) Riveted Aluminum Alloy; Three Stiffeners											
0.2	0.23	0.30	0.38	0.46	0.54	0.63	....	....	....	....	....	....
0.3	0.34	0.45	0.55	0.65	0.77	0.88	....	....	....	....	....	....
0.4	0.44	0.56	0.69	0.82	0.95	1.08	....	....	....	....	....	....
0.5	0.52	0.66	0.81	0.96	1.10	1.24	....	....	....	....	....	....
0.6	0.60	0.75	0.91	1.08	1.23	1.39	....	....	....	....	....	....
0.7	0.66	0.83	1.01	1.18	1.35	1.51	....	....	....	....	....	....
0.8	0.72	0.91	1.10	1.28	1.46	1.63	....	....	....	....	....	....

$\times 3 \times \frac{5}{16}$, with the $3\frac{1}{2}$ in. leg facing the web. The radius of gyration of this angle about the face of web is 1.69 in., which is in excess of the required value.

Summary

The paper discusses the basic laws governing the buckling of metal elements which form structural members. The behavior of these elements, subjected

to various stress conditions and combinations, is analyzed. The results are applied to unstiffened plates of columns and girders. The studies are then extended to structural parts reinforced by stiffeners. The analysis leads to general expressions which enable the engineer to judge and weigh the action and behavior of stiffened plates. These expressions are applied to column plates and webs of plate girders reinforced by stiffeners. An extensive study of the behavior of stiffened plates in columns and girder made it possible to establish simple rules for practical design. Tables and diagrams have been prepared from them to facilitate the work.

Conclusions

The study of the elastic stability of metal elements given in the paper has clarified the general behavior of buckling. It has made possible the establishment of simple rules for the design of structural members reinforced by stiffeners. The application of these rules results in more dependable and more economical structures. It gives greater freedom to the structural application of metals and will lead to simpler fabrication and erection.

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PAPERS

CHICAGO RIVER CONTROL WORKS

By H. P. Ramey,[1] M. Am. Soc. C. E.

Synopsis

The Chicago River Control Works constructed in 1939 by The Sanitary District of Chicago (Ill.) prevent the polluted Chicago River from entering Lake Michigan. They consist of walls making a watertight enclosure around the river mouth, a navigation lock to pass shipping, and two sets of control gates to regulate the quantity of water admitted into the river from the lake (see Fig. 1). The westerly or river end of the lock is approximately 1,700 ft

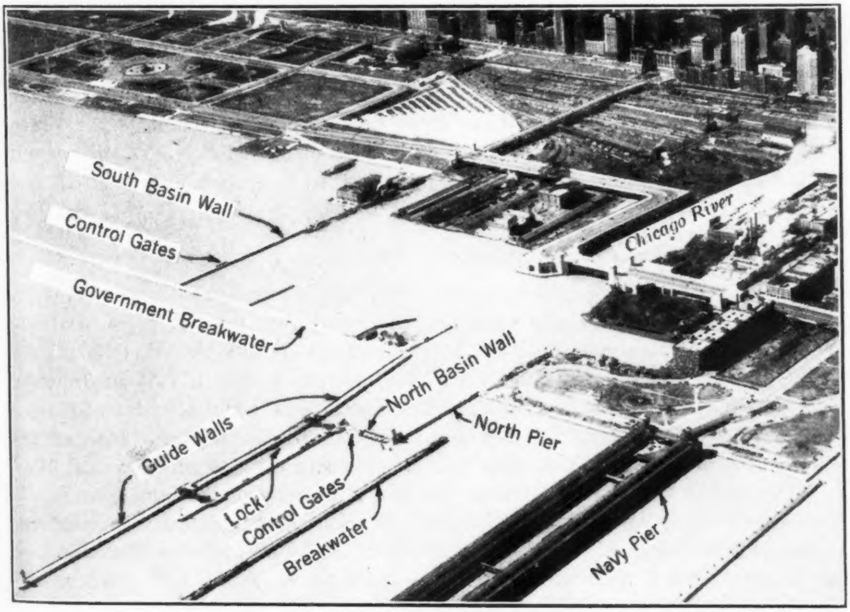

Fig. 1.—General View of Chicago River Control Works

Note.—This paper was presented at the meeting of the Waterways Division, San Francisco, Calif., July 26, 1939. Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by **May 15, 1940**.

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east of the Outer Drive Bridge, which crosses the Chicago River at its mouth. The longitudinal axis of the lock extends from east to west, in line with the river. The south wall of the lock is extended 700 ft easterly of the east lock gate, to serve as a guide wall and breakwater. From the northwest corner of the lock a north basin wall runs north, connecting with an existing United States North Pier which in turn connects with the shore. In this wall are four control gates. From the southwest corner of the lock a guide wall extends westerly and slightly southerly, connecting with the northerly end of the United States breakwater. This breakwater extends due south, and from a point 1,600 ft south of its connection with the guide wall, a south basin wall was constructed to the west, connecting this breakwater with the shore. Four other control gates are in this south basin wall. The two basin walls, the lock, and the southwesterly guide wall, in conjunction with the original United States breakwater and North Pier, enclose an inner harbor basin about 1,700 ft by 2,000 ft.

Many novel features were embodied in the design of the lock gates and walls and these are discussed herein, as well as the purpose of the Chicago River Control Works.

History

The purpose of the Chicago River Control Works is to separate, definitely and effectively, the Chicago River from Lake Michigan, which is the source of municipal water supply for Chicago and its environs. A detailed account of the history of this project has been presented by Samuel N. Karrick, M. Am. Soc. C. E., Major, Corps of Engineers, U. S. Army.[2]

General Conditions

In this paper all elevations are referred to Chicago City Datum (C.C.D.) which is 579.94 ft above mean sea level. The average level of Lake Michigan is about 581.00 ft above mean sea level, or about + 1.00 C.C.D. Its highest level, monthly mean, was 584.69 in 1838, and its lowest 577.35 in January, 1926. This indicates a range of 7.34 ft. The level of Lake Michigan averages about 1.00 ft higher during the summer than in the winter. It averaged about El. − 1.50 C.C.D. during the construction seasons of 1936 and 1937.

The bed of the lake in Chicago Harbor at the site of the lock (see Fig. 2) has an elevation of about − 25.0 C.C.D. Soil tests, made by driving test piles, indicated soft blue clay for a depth of 20 to 30 ft below the lake bed, or from − 25.0 C.C.D. down to El. − 45.0 to − 55.0 C.C.D., where stiff blue clay was encountered.

The lock walls and harbor enclosure walls were designed to sustain a head of 8 ft of water in either direction. The river level may be higher than the

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[2] "Protecting Chicago's Water Supply," by Samuel N. Karrick, *Civil Engineering*, September, 1939, p. 547.

lake, during flood conditions, but ordinarily the lake level will be the higher. In the future it may be desirable to hold the river level at some fixed elevation, such as − 2.00 C.C.D., to provide definite vertical clearance under bridges across the Chicago River, under which conditions the differential in water levels may average 2 ft. At present this differential is about 6 in., or less.

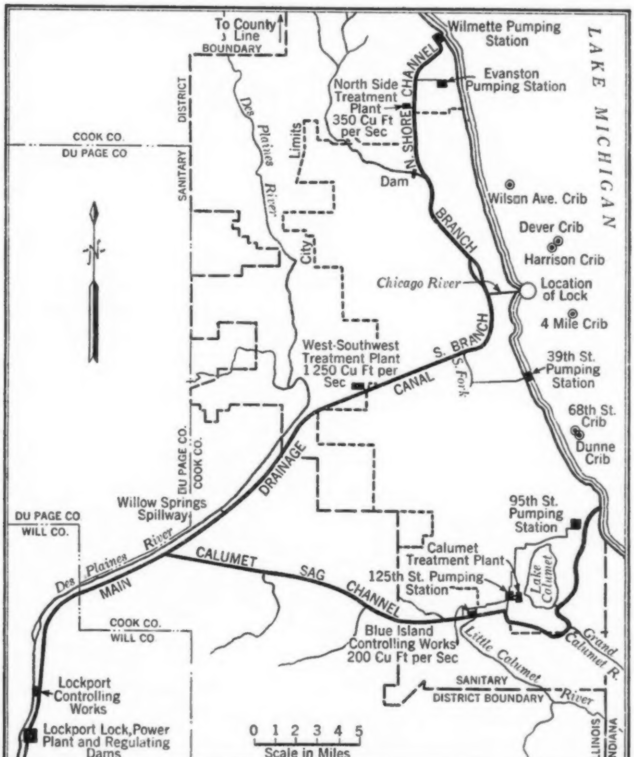


Fig. 2.—Intake Cribs and Channels, Sanitary District of Chicago

It was necessary that the lock gates sustain a head of water in either direction, and that they remain closed, against surges and heavy seas from the lake, even when the static head might be very small. This eliminated the use of ordinary miter gates.

It was desirable that the control gates be stable, at any width of opening, in order to obtain reasonably close control of the flow from the lake into the river. It was desirable also that gates be installed as flood gates, to close automatically and prevent any flow whatever from the river into the lake so as not to place any reliance upon human judgment as to when the gates should be closed in case the river level became higher than the lake. This problem was solved by installing sluice gates for control, and flap gates on the inner side of the gate structures for protection.

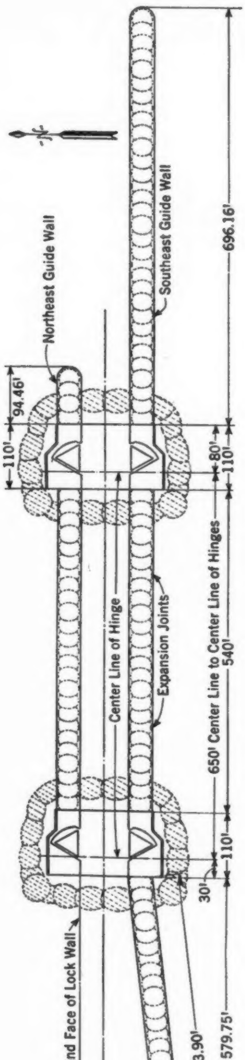
To have constructed the Chicago River Control Works in accordance with designs ordinarily used for such structures (lock walls of concrete supported on piles, miter lock gates, standard lock valves, filling conduits, etc., and connecting walls of rock-filled piers capped with concrete) would have cost about \$4,800,000 or 60% more than the \$3,000,000 of funds available. To effect a 60% saving in cost, it was obvious that there must be a radical departure from the orthodox method of lock and wall construction. There is no known way to construct lock gates, or their supporting structures, inexpensively. Such construction must be permanent in nature. The lock walls, guide walls, and the walls connecting the lock with the shore formed the major part of this particular job and it was in wall or pier construction that the principal saving was effected.

WALLS

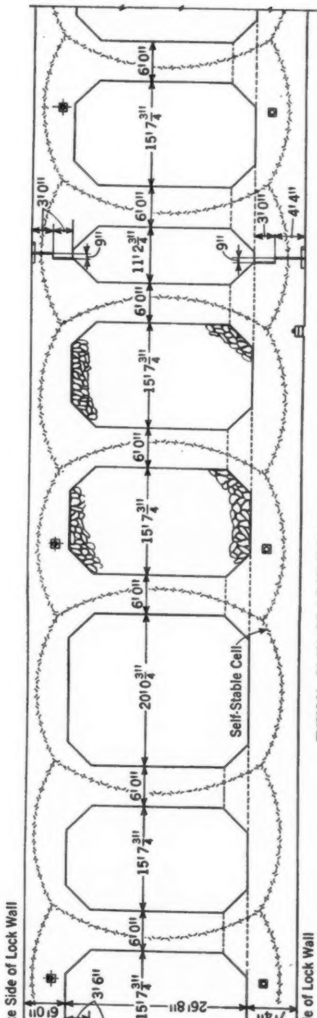
In recent years, numbers of breakwaters and piers have been constructed using a series of rock-filled circular cells formed by driving interlocking steel sheet piling. The retaining members are all in tension, effecting a considerable saving in structural material. In many places such cells have been filled with gravel and sand; and, for cofferdams so built, clay has proved a good filling medium. Such structures are slightly flexible and a concrete cap increases rigidity. This circular cell type of construction was adopted for all the walls in this project, purely in the interest of economy (see Fig. 3). Since lock walls of this type would not permit the construction of filling and emptying conduits, it became necessary to provide such facilities at the ends of the lock. The two massive concrete gate foundation structures, one at each end of the lock, and the two control gate sections, one in the north basin wall and one in the south basin wall, also of concrete, were built "in the dry," inside cofferdams. Except for these structures and for the floor of the lock all of the construction, being wall construction, was performed "in the wet" and without the use of divers. The four concrete structures were constructed in definite locations and tied together by the cellular type walls.

Lock Walls.—The characteristic dimensions of the lock walls are given in Figs. 3 and 4. A radius of 19 ft $2\frac{1}{4}$ in. was used for the arc forming each inner and outer face of each cell. A radius of 34 ft 1 in. was used for the arc forming the diaphragm between contiguous cells. To enable work at more than one place on each wall, the cells were divided into four groups in each wall. The piling for the center cell of each group, or the initial cell, was driven first, followed by two cells on each side of the center cell. This left elongated cells, in which the diaphragms curved inward, to join the various groups together. Steel sheet piles, 58 ft long, were used and were driven to a bottom elevation of − 55.0 C.C.D., giving a minimum penetration of 30 ft in the clay and leaving the tops of the cells at + 3.0 C.C.D. Special **Y**-piles were fabricated by riveting together half sections of regular sheet piles to form the junction between contiguous cells (see Fig. 3).

The main consideration in the steel sheet piling, as used, being always in tension, was strength in the interlocks. A steel was specified soft enough to



ARRANGEMENT OF STEEL SHEET-PILE CELLS, ALSO OF COFFERDAM CELLS FOR GATE BLOCKS



TYPICAL PLAN OF LOCK WALL

FIG. 3.—PLAN OF CELLULAR TYPE WALLS

withstand the strain and distortion due to driving but hard enough to maintain rigidity once it was in place. The web thickness was $\frac{3}{8}$ in. and the width from point of interlock to point of interlock 16 in., with a tolerance of $\frac{1}{8}$ in. The phosphorous content and the sulfur content were each limited to 0.06% maximum and the tensile strength to a minimum of 70,000 lb per sq in. Bend test specimens were required to withstand bending 180° around a pin with a diameter twice the thickness of the specimen, without fracture on the outside of the bend. Three tests of pile sections were made of each heat of steel and the average interlock strength of the three tests was required to be not less than 18,000 lb per lin in., with no one test showing an interlock strength of less than 15,000 lb per lin in.

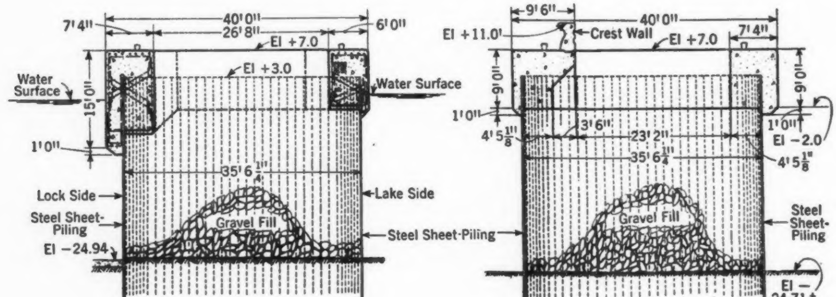


FIG. 4.—CROSS SECTIONS OF CELLULAR TYPE WALLS

No failure of any pile occurred in the permanent work. During construction, one failure occurred in a temporary **Y**-pile forming the junction between two inner walls of two cells of the cofferdam around the west gate foundation structure of the lock. Failure occurred in the web of the pile and was ascribed to exceptionally great stress caused by the use of a long radius in the arc of the inner curved surface of the cofferdam cell and the omission of weep holes in the piling. The cofferdam cells had no concrete cap. The radius used in this cell was 34 ft, compared to a radius of 19 ft $2\frac{1}{4}$ in. used for the cells of the lock wall. The maximum radius allowable was 28 ft. The accident was really caused by a boat striking the outer wall of the cofferdam.

The cells were filled, to El. − 2.0 C.C.D., with broken rock, graded in sizes from $\frac{1}{4}$ in. to 20 in. From this level, to El. + 7.0 C.C.D., the top of the walls, the structure was filled with gravel, 95% of which would pass a $\frac{3}{4}$-in. screen.

Rock was preferred as a medium for filling the cells, principally because of the shear that could be developed in a pile of stone, compared to that of any other medium. Clay would have been least expensive, as it was at hand, but it would have developed no shear and it could dissolve and escape through any hole or fissure. Sand would have assumed its final position quickly but it would have developed no shear and fine sand could have escaped through a hole or fissure. Gravel had the quality of assuming its final position quickly

but of developing no shear. Stone had one drawback—namely, that it would gradually settle into the soft clay inside the cell at the bottom. This settlement was considered less serious than the faults inherent in the other materials. Alternate bids were taken for gravel fill and rock fill and the successful bidder submitted a price of \$1.80 per cu yd for gravel fill, and \$2.25 per cu yd for rock fill.

The gravel specified was to have at least 30% igneous material. It would have been of a quality suitable for use in concrete. The sizes were:

95% to 100% passing a $\frac{3}{4}$-in. screen
80% to 95% passing a $\frac{1}{2}$-in. screen
15% to 20% passing a $\frac{1}{4}$-in. screen
0% to 5% passing a No. 10 screen

The stone specified was to be hard and durable, weighing, solid, not less than 160 lb per cu ft, and, as furnished, not less than 95, nor more than 105, lb per cu ft. The stone, as graded, was to vary from $\frac{1}{4}$ in. to 20 in. along greatest dimension, with not more than 5% passing a No. 10 screen. The stone was to be placed with a belt conveyer.

Rock fill was used for the cells of the lock walls below El. − 2.00 C.C.D. the approximate low-water level, and gravel fill was used for the upper 11 ft of the structure. Gravel fill was used for the cells of the guide walls. Certain economies were effected and construction was expedited by using gravel in the guide walls, since the same gravel could first be used in the cells of the cofferdams around the lock-gate foundation structures, and it was easy to dig the gravel from the cofferdams. These cofferdam cells also furnished convenient storage bins for gravel for concrete.

A concrete cap was constructed on top of the cells, formed of a grid of heavy concrete beams, each beam embedding the upper part of a cell face or a cell diaphragm. This concrete cap extended from El. + 7.0 C.C.D., down 16 ft, to El. − 9.0 on the inside of the lock, and down 10 ft to El. − 3.0 on the outer or lake side of the wall. Inside the cells, this concrete extended down 13 ft on the lock side of the cells, and 9 ft at the diaphragms and at the lakeward side. The width of the beam forming the face of the lock was 7 ft 4 in., most of the other beams being 6 ft wide. Holes were burned through the webs of the sheet piles where they were embedded in concrete, to permit the placing of reinforcing rods to tie together the bottoms of the concrete beams where there was a separation due to the sheet piling.

This system of concrete beams, forming a grill laid down over the rock-filled cells, provided a walk 7 ft 4 in. wide along the inner side of the lock wall and a walk 6 ft wide along the outer side, with cross walks 6 ft wide. This left open spaces over each cell, 26 ft 8 in. by 22 ft 8 in. over the initial cells and 26 ft 8 in. by 15 ft 7 in. over the remaining cells. Gravel, filling the upper portions of the structure, was leveled off at El. + 7.0 in these open spaces. Originally, a concrete slab, with several manholes through it, was planned, in place of these open spaces, to give a better footing than the loose gravel. Some settlement of the rock and gravel fill inside the cells is expected and when

that has occurred these open spaces will undoubtedly be covered with a suitable floor. Three 1-in. mastic expansion joints were provided in the concrete cap of each lock wall, or at average intervals of 135 ft.

The advantage of the open concrete grill on top of the rock-filled cells, acting as a horizontal truss, over the same grill stiffened by a concrete slab is not apparent to many structural engineers. The difference in action, if any, should be very slight.

The lock wall was designed as a gravity dam, to sustain a head of 29 ft of water, a condition possible only if the lock should be empty and the water level on the outside at El. + 4.0 C.C.D. In addition, a horizontal thrust of 5,500 lb per lin ft of wall was assumed to be applied at El. + 5.5, to equal the force of a 6-ft wave. It was assumed that the cell would be drained empty of water by weep holes through the piling on the lock chamber side but that seepage through the lake side would decrease stability, to equal which, uplift was computed as a triangle at zero on the lock side and half the hydrostatic pressure on the lake side. Rock and gravel fill was computed at 100 lb per cu ft and the concrete cap at 150 lb per cu ft, these supplying the vertical downward force. Friction between the rock fill and the sheet piles, pulling resistance of the lake-side piles and supporting resistance of the lock-side piles were all disregarded. These assumptions resulted in the following total forces per linear foot of wall:

Direction	Forces, in pounds
Vertical downward	131,400
Vertical upward	15,800
Horizontal	32,900

The resultant of these forces falls slightly within the middle third of the assumed 34-ft wall base at El. − 25.0 C.C.D.

The horizontal pressure inside the cell at the base, due to the rock or gravel fill, was estimated at 2,240 lb per sq ft, with the lock empty and with no water in the cell. The direct unit tensile stresses in the steel sheet piles were computed as:

Location	Tensile stress
In the web (pounds per square inch):	
Of lock side sheet piles	9,600
Of diaphragm sheet piles	11,700
Of lock-side fabricated Y-piles	13,400
Of diaphragm fabricated Y-piles	16,200
In the interlock (pounds per linear inch):	
Of lock-side piles	3,580
Of diaphragm piles	4,300

The sheet piles were assumed to carry the entire weight of the reinforced concrete cap, producing the following vertical loads on the piles:

Type of pile	Load, in pounds per pile
Lock side	21,100
Lake side	13,700
Diaphragm	13,400

The supporting power of one pile, with a skin friction of 500 lb per sq ft and 30 ft of penetration, would be 40,000 lb.

No particular difficulties were encountered in the construction of these lock walls. A template was floated in the water, around which the piles were driven. Since the cells were formed in groups, work could be done at several places at one time. Because of possible distortion of the cells, when empty, by wave action, it was desirable to fill them with stone as quickly as possible. It was always advisable to have a cell empty when piling was being driven for its contiguous cell. The first cell was then more flexible and more easily permitted the driving of the final closure pile. The exposed portion of the steel sheet piles is permanently below water level, the top portion being encased in concrete down to the low-water line. The use of copper bearing steel for the piles was considered but the additional protection, in increased length of life of the piling, was not deemed worth the added expense.

Protection of Lock Walls.—No protection was given to the concrete in the walls of the lock from impact or rubbing by boats. Horizontal wales of either steel or wood were suggested, to be attached to the concrete inner faces of the lock walls, but were ruled out, one argument being that the edge of a small boat might get caught under a wale and the boat tilted. A suggestion that wooden wales be used to cap a row of wooden piles along each of the inner sides of the lock walls met with no favor because this would have decreased the usable width of the lock by approximately 2 ft. Floating fenders were also considered.

Use of the lock for less than one season has demonstrated the need for more care on the part of operators of boats or of some protection to the concrete if spalling is to be avoided. Large boats striking the walls have spalled off considerable pieces of concrete, particularly at the angle between the top and inner face of each wall. There has also been considerable spalling at the sides of certain vertical ladder recesses. None of the spalling has penetrated to any great depth and the stability of the walls has not been, and will not be, in any way impaired, but ultimately a ragged appearance will be created unless the impact of boats is avoided in some manner. It is largely a matter of judgment as to whether the expense of protective devices is justified or whether it is better to assume the cost of repairs.

Guide Walls.—Three guide walls were provided, using the same design and dimensions as for the lock walls, except that the concrete cap was extended down only 10 ft, to El. − 3.0 C.C.D., on both sides of the guide walls, or similar to the outer side of the lock walls.

The northeast guide wall is 94 ft long and serves only to aid in steering incoming boats into the lock. The southeast guide wall is 696 ft long and serves as a breakwater, as well as for mooring purposes. The southwest guide wall, 613 ft long, also serves as one of the enclosing walls of the inner harbor basin.

No special design was made for the guide walls. They are subject to impact from vessels in motion and to impact from waves, and a reasonable wall width was necessary. The width of 40 ft, as used in the lock walls, was maintained for uniformity as well as for ample stability.

For the most part, the easterly guide walls were constructed with sheet piling that previously had been used in forming the cofferdams around the concrete gate block structures during construction. Practically all of the cells of the guide walls were filled with gravel which previously had been used in the cofferdams around the gate blocks.

North Basin Wall.—The north basin wall is 216 ft 6 in. long, from the north face of the westerly gate block of the lock to the Government north pier, including the control gate section 59 ft long. The part of this wall of the cellular type is 157 ft long, and its design and construction is similar to that of the guide walls. The cells were filled entirely with broken rock ranging in size from $\frac{1}{4}$ in. to 20 in.

The reinforced concrete cap of the north basin wall is similar to the concrete grid over the guide walls, with the addition of a 24-in. solid concrete slab over the entire top, 40 ft wide, poured integrally with the concrete frame or grid. This also serves as the roadway approach to the lock, and a traffic lane, 9 ft wide, is kept clear of any obstruction.

South Basin Wall.—The south basin wall is 1,181 ft long, from the shore of Lake Michigan to the Government breakwater, including a concrete section, 59 ft long for four control gates. The water depth averages less than 20 ft and the width of the wall is 24 ft. This wall is of the straight-faced type, being formed of two rows of steel sheet **Z**-piling, retaining a rock fill. A stretch of 457 ft of this wall was constructed of steel sheet piles 47 ft long and the remainder of piles 55 ft long. All piles were driven until a uniform top elevation of + 5.75 C.C.D. was attained, giving an average penetration in the soil of about 25 ft.

A cap, formed of a 15-in. by 33.9-lb channel, was welded to the top of each row of **Z**-piles. At El. − 0.50 C.C.D. two 12-in. by 30-lb channels were bolted to the inside of each row of the face piling. Tie rods $2\frac{3}{4}$ in. and $2\frac{7}{8}$ in. in diameter passed through the face piles and through these channels, tying the two rows of **Z**-piles or face piling together. Two 80-lb railway rails were welded to the outside of each row of the face piles, one at El. + 1.33 C.C.D., and the other at El. + 4.83. These serve as wales or guard rails. At intervals of 84 ft, diaphragms formed of steel sheet piling 34 ft long driven to a top elevation of + 4.00, across the wall, divide this wall into a series of rectangular pockets. The tops of the piles in each diaphragm are embedded in a reinforced concrete cross beam, 3 ft 6 in. wide by 2 ft 6 in. deep, at each end of which is anchored a cast steel mooring post.

This wall was filled to El. + 5.50 C.C.D. with broken rock ranging in size from $\frac{1}{4}$ in. to 20 in. Above this level was placed a 6-in. crushed-stone top finished with limestone screenings. This roadway top was given a 2-in. crown at the center.

The south basin wall was designed as a gravity dam, to sustain water pressure due to a head of 7.83 ft, from El. + 5.83 C.C.D., to El. − 2.00. The base of the wall was assumed at El. − 26.00 C.C.D. and the width at the base, 23 ft. No wave action and no surcharge was assumed. Weight of rock fill was assumed at 90 lb per cu ft "in the dry" and at 65 lb per cu ft submerged. The face piles were designed as vertical, simple beams with a

cantilever at the top, to carry the horizontal load caused by the submerged rock fill and a head of 7.83 ft of water, the points of support being assumed at El. − 0.50, the elevation of the tie rods, and at El. − 30.11, the calculated first point of contraflexure below the lake bottom.

Lock Gates.—Fig. 5 is a view of the lock gates. The requirement that these gates be capable of withstanding a head of water in either direction and also remain closed against surges from the lake eliminated ordinary miter gates from consideration. A modified form of sector gate was adopted. The word "sector," as used in this paper, applies to the shape of a horizontal cross section through the prism (one sixth of a circular cylinder) forming one leaf of a set of lock gates. The sector gate, as a lock gate, had previously been developed by the engineers of The Sanitary District of Chicago and had been used as such in the controlling lock at the head of the Calumet-Sag Channel, near Blue Island, Ill. This sector gate had been developed from the idea of the sector dam used to control flow in the Main Drainage Channel at the Lockport Power House and it, in turn, had come from the idea used in the bear-trap dam at the old controlling works at Lockport, Ill. This bear-trap dam, properly counterweighted or supported by water pressure, inside or beneath the dam, was stable in any position. The same was true of the sector dam at the Lockport Power House, which was placed with its curved face upstream and, with a tight sheet-steel top, was controlled in any desired position by merely varying the pressure of water inside the dam.

The sector gates at the Blue Island lock had a tight sheet-steel curved (circular arc) face on the upstream side. All stresses were carried to the hinge, at the center of the circle, through framed members. Since these pressures all acted radially each gate leaf was stable in any position, and flow through the lock and through the Calumet-Sag Channel was controlled by setting the gates in any partly opened position. Filling and emptying valves were provided in the Blue Island lock for use when it was performing its function as a navigation lock. The U. S. Army Engineers had used the same idea in lock gates in some of the canals leading from Lake Okeechobee, in Florida. These locks were used for both navigation and flood control. The Army Engineers had gone a step further and had omitted filling and emptying valves and filled these locks by merely opening the upstream gates and emptied them by opening the downstream gates. Tests indicated that this could be done without danger to water craft in the lock at the time.

The foregoing considerations led to the adoption of sector gates for the lock at the mouth of the Chicago River and the omission of filling and emptying valves. The lock at Blue Island was 50 ft wide by 400 ft long, and the travel of each leaf of each set of the sector gates there was 25 ft. The locks in the canals in Florida were even smaller. The Chicago River lock, being 80 ft wide by 600 ft long, involved greater quantities of water for filling and a travel of 40 ft for each leaf of the sector gates. This consideration led to the adoption of triangular shaped gate leaves, giving three passages for the admission of water to the lock when the gates were in a partly opened position, in place of the one central passage possible if true sector gates had been used.

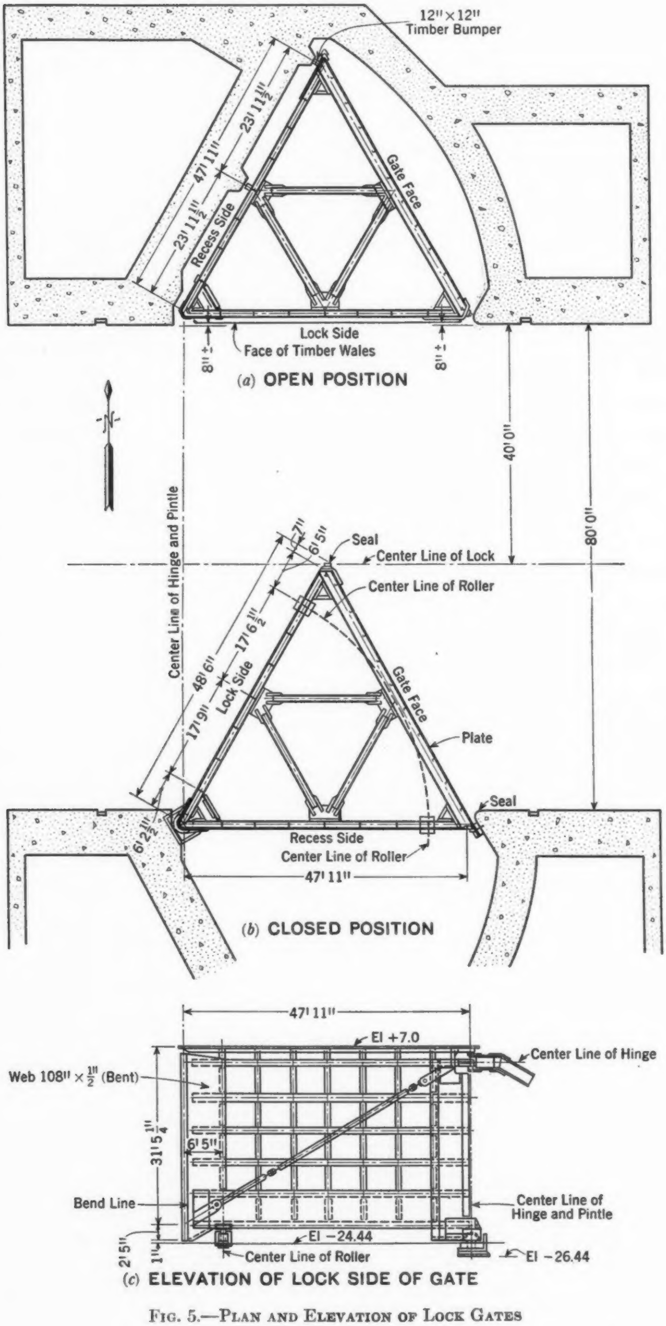


FIG. 5.—PLAN AND ELEVATION OF LOCK GATES

Each gate leaf is a vertical triangular prism, 48 ft on each side and 30 ft 6 in. high. In an open position, each fits into a recess at the side of the lock, in the massive concrete gate block. The shape of this recess is that of one sixth of a circular cylinder. The framing of each gate leaf consists of six horizontal structural steel triangular frames, with vertical girders at the three corners of the prism. The members of the horizontal frames and vertical posts form the three vertical sides. The side of the prism forming the face of the gate leaf is covered with a tight $\frac{3}{8}$-in. steel plate. The other sides are open framing, with vertical and horizontal members only (see Figs. 6 and 7). A diagonal tension rod, provided with turnbuckles, extends from the top of each of these open sides at the hinged end to the bottom of these open sides at the face end. The face of the gate is toward the lake. The triangular gate-leaf structure is supported at the bottom corner opposite the gate face on a 26-in. hemispherical cast steel pintle, over which is fitted a properly shaped phosphor-bronze bushing, which in turn is attached to the framing of the gate leaf. This forms the lower hinge. The upper hinge, at the top of the structure, consists of a ball-and-socket joint, one member of which is anchored solidly to the concrete and the other attached to the framing of the gate-leaf structure through an adjustable screw.

A flat steel track, curved in a circular arc, with a radius of 41 ft 6 in., is set in the concrete below the gate framing to carry two rollers attached to each gate-leaf frame. Distortion of the entire framed structure can be prevented by means of the diagonal, turnbuckle, tension rods; and, by means of the adjustable screw at the top hinge, the weight can be distributed between the hinge and the rollers in any proportion desired.

Five horizontal timber wales are bolted to the lock side of each gate-leaf structure and two vertical timber bumpers are bolted to the recess side, at effective points. Checkered steel plate walks and handrails are provided on all three sides of this structure, at the top. When the gates are closed these walks afford passage from one side of the lock to the other. Rubber sealing strips are provided at the bottom and the two vertical sides of each gate-leaf face.

Operation of each gate leaf is by means of a strut, one end of which is hinged to the upper horizontal frame of the structure. Attached to one side of this strut is a rack which meshes with a pinion driven by an electric motor.

A flat steel track, 13 in. wide and 3 in. deep, curved in a circular arc with a radius of 41 ft 6 in., is set in the concrete below the gate framing to carry the rollers attached to each gate-leaf frame. Each roller is of cast steel 20 in. in diameter and 10 in. long, slightly beveled to run on the horizontal curved track. It is finished on both its rolling surface and on its inside to receive a phosphor-bronze bushing. The bushing fits around an 8-in. diameter pin of cold-rolled steel which is finished all over. The pin is keyed to a heavy structural steel frame which in turn is rigidly attached to the bottom member of the frame of the gate leaf. The bushing is provided with grease grooves which are fed from holes to the center of the pin, where a central hole extends to one end and is connected to a $\frac{1}{4}$-in. hard drawn copper tube that extends up to the

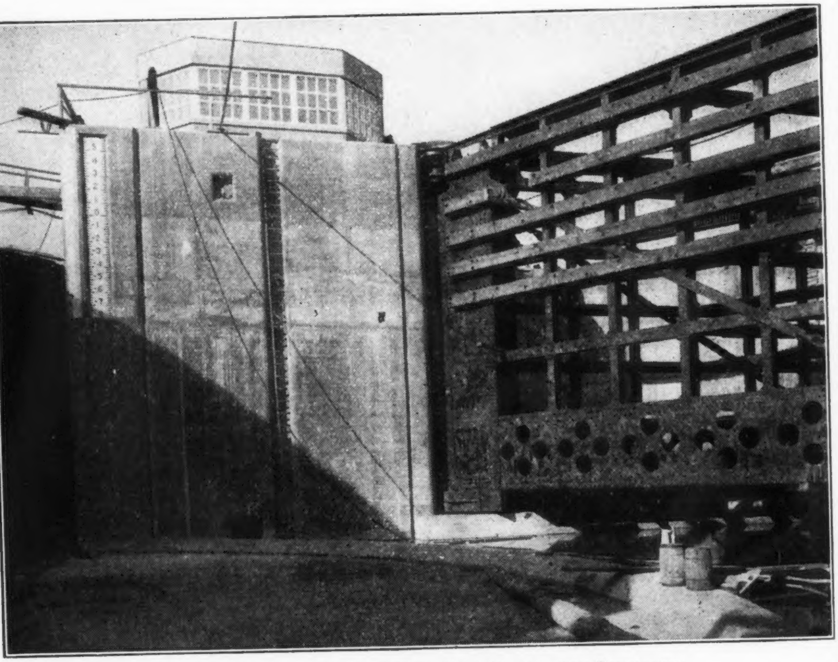


FIG. 6.—VIEW FROM SILL OF WEST GATE FOUNDATION STRUCTURE

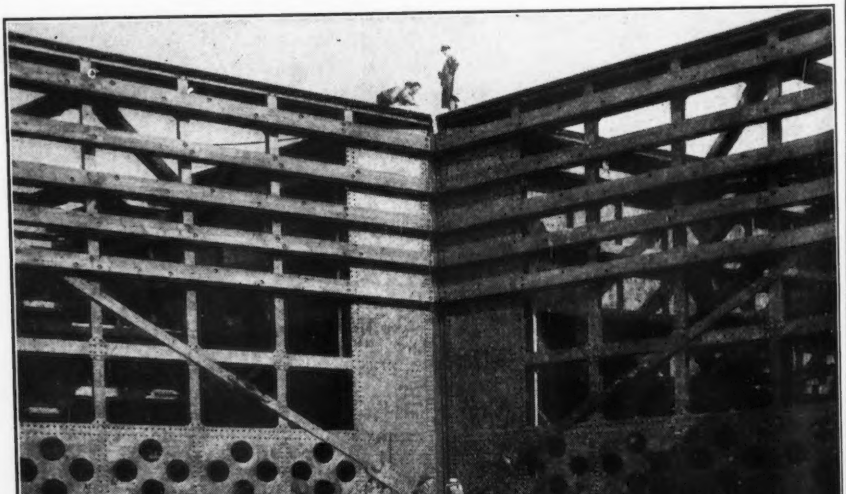


FIG. 7.—VIEW OF WEST LOCK GATE IN CLOSED POSITION DURING CONSTRUCTION

top of the gate-leaf frame, terminating in a special fitting. A flat wire push broom is installed in the front and rear of each roller to sweep the flat steel track clear of debris. The concrete curb in which the steel track is embedded is raised approximately 2 ft above the adjacent lock floor.

The hemispherical pintle forming part of the lower hinge, the ball-and-socket joint forming the upper hinge, and the two diagonal turnbuckle tension rods on the two downstream sides of each gate leaf enable adjustment of this framed structure to any position and to any extent which, conceivably, may ever be needed. To date (1940) the adjustment of the lock gates has been such that 80% of the weight of each gate leaf has been on the rollers and the operation of the gates has been extremely smooth. This lock is operated on an average of more than 50 times daily during the open navigation season. Each of the four gate leaves weighs 101 tons and is operated easily by two motors, the combined power of which amounts to only $26\frac{1}{2}$ hp.

The maximum horizontal load, due to a head of 8 ft of water, is 250,900 lb on the upper hinge and 362,200 lb on the lower hinge. The maximum vertical load on a roller, due to the weight of a gate leaf, is 83,400 lb, and on the pintle is 41,800 lb. When the rollers are not functioning the upper hinge receives an additional horizontal load of 204,300 lb, and the lower hinge receives an additional horizontal load of 204,300 lb and an additional vertical load of 161,000 lb. The total maximum loads on the hinges are, therefore:

Upper Hinge	
Horizontal load	455,200 lb
Vertical load	0
Lower Hinge	
Horizontal load	566,500 lb
Vertical load	202,800 lb

The foregoing assumed that loads give maximum bearing pressures of 1,720 lb per sq in. on the upper hinge bushing and 1,410 lb per sq in. on the lower hinge bushing.

Important features of the gates in the lock at the mouth of the Chicago River are: (1) Simplicity of fabrication because of the flat gate face and the use of few built-up members in the frame work; (2) the ability to adjust the gate leaf and remove distortion; (3) the equilateral triangular shape of the gate leaves makes them self-balancing horizontally; (4) the design of the gate hinges is such that no binding can occur; (5) the operating strut and rack and pinion are all above high water; (6) and the triangular shape of the gate leaf, with the sector shape of its recess, permits water to flow around both ends of the gate face when water levels are being equalized.

Concrete Gate Blocks.—The concrete gate blocks at each end of the lock were built "in the dry," inside of cellular type cofferdams (see Fig. 8). The foundation of each of these structures consists of a heavily reinforced concrete floor, supported on wooden piles. The dimensions of this floor slab are 110 ft longitudinal with the axis of the lock by 196 ft 6 in. transverse to the lock, with a thickness of 7 ft, except for the central 40 ft and for the part under the gate recesses where the thickness is reduced to 4 ft. Each of these heavy floor

slabs is supported on 1,632 wooden piles, 27 ft long under the 7 ft thick section and 30 ft long under the 4 ft thick section, driven to El. − 59.00 C.C.D. on a maximum of 4-ft centers. The piles project up about 9 in. into the concrete floor. Steel sheet piles extending up 1 ft into the concrete slab, 1 ft 6 in. from the outer edges, completely surround the wooden piles and prevent the soil from squeezing out. These sheet piles were driven to El. − 45.00 C.C.D. This concrete floor slab was reinforced principally as a beam, spanning the soil across the lock. The top of this floor was made at El. − 26.44 C.C.D.

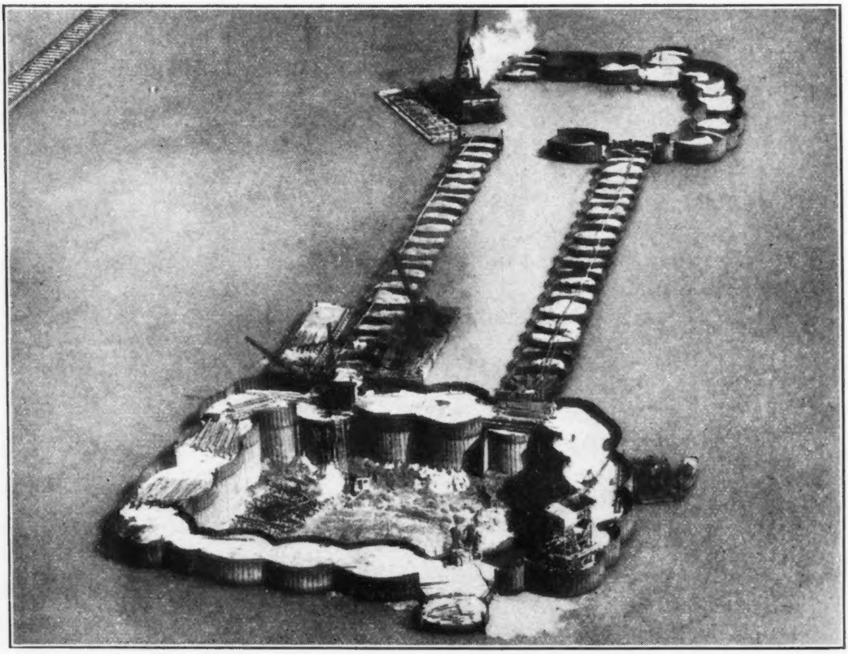


FIG. 8.—GENERAL VIEW OF WALL CONSTRUCTION AND COFFERDAM FOR CONSTRUCTION OF CONCRETE GATE BLOCK "IN THE DRY"

On each side of the lock a concrete gate block was constructed, 110 ft long and 53 ft 9 in. wide from El. + 7.00 C.C.D., down to the floor slab at El. − 26.44. A 60° sector shaped gate-leaf recess, with a radius of 52 ft 6 in., was formed in each of these gate blocks. On the floor slab between the gate blocks a V-shaped curb, 2 ft high and 4 ft wide, was constructed as a seal for the lock gates when in a closed position. Two curved curbs were also constructed, 1 ft high and 2 ft 3 in. wide, one upstream and one downstream from each lock gate, to serve as bottom guides and supports for cofferdams in case it should be desirable to dewater the space around the gates for repairs. A similar straight curb was constructed along the outer edge of each gate-leaf recess to serve as a guide and support for a straight cofferdam in case it should be necessary to dewater a gate recess. Two curved track foundations were

also built on the floor slab, embedding heavy track plates at El. – 24.52 C.C.D., on which the rollers of the gate leaves operate.

Each gate foundation structure was designed in various ways: (*a*) As a rigid frame, with the entire structure "in the dry" and the gate leaves in their recesses (a condition occurring only during construction); (*b*) with the entire structure submerged in water to El. + 4.00 C.C.D., and with wave force of 6,000 lb per lin ft applied to the side of one gate block at El. + 4.00; and (*c*) with the two curved temporary cofferdams in place, the gate space across the lock, dry, with surrounding water at El. + 4.00, full hydrostatic uplift under the entire base of the floor slab and no bond between the supporting piles and the concrete slab.

Each gate block was designed as a rigid frame: (*a*) With the walls of the gate block hinged in the floor slab at El. – 28.44 C.C.D.; (*b*) with the walls of the gate block fixed in the floor slab at El. – 28.44; and (*c*) with a gate recess cofferdammed and pumped dry, with surrounding water at El. + 4.00 and a force from wave action and boats applied at El. + 4.00.

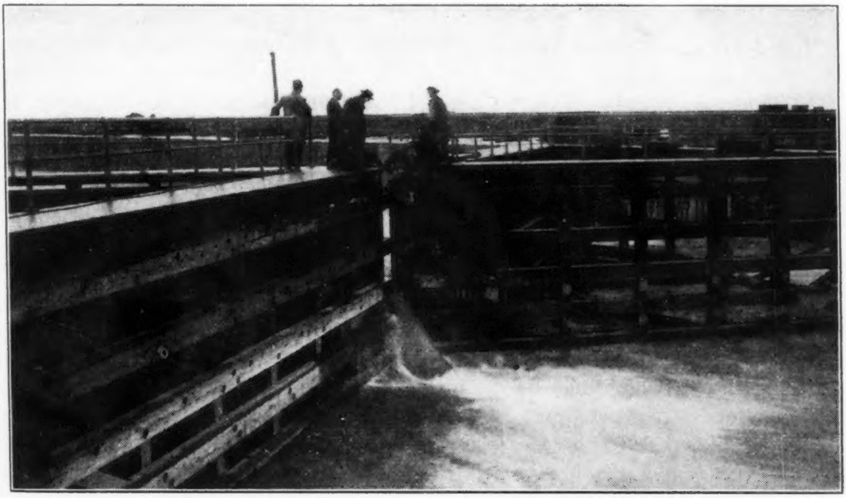


FIG. 9.—VIEW OF EAST GATE BEING OPENED DURING A TEST UNDER 8 FT OF DIFFERENTIAL HEAD

Operating Machinery.—The lock is filled or emptied by opening one gate slowly. Fig. 9 shows the east gate being opened during a test under a differential head of 8 ft. The operating machinery for each lock gate leaf consists of two electric motors mounted in conjunction with a suitable speed reduction unit. A small motor (1.5 hp and 1,145 rpm) is used for low-speed operation at the start and at the finish of each movement of the gate leaf. A large motor (25 hp and 1,160 rpm) is used for high-speed operation. A speed reduction of 30,000 to 1 is made (in four steps) between the pinion on the output shaft of the small motor and the operating pinion meshing with the rack on the operating strut hinged to the gate leaf. A speed reduction of 350 to 1 is made

(in two steps) between the pinion on the output shaft of the large motor and the operating pinion. The large motor can be cut in or out while the small motor is still operating. A limit switch automatically cuts off the current from the smaller motor when the gate leaf has reached its limit of travel either in a closed or an open position.

To open the gates against no head of water, the operating pinion is started at high speed (both motors running) and operated at this speed until the gate leaf is within a few inches of its fully open position, when the large motor is cut out automatically, the small motor continuing operation until the gate leaf is in its fully open position, when the limit switch cuts out the small motor. In the operation of closing the gates the procedure is almost identical; the operating pinion runs at high speed until the gate leaf is within a few inches of its closed position, when the large motor cuts out, the gate being moved on, at slow speed by the small motor, to its fully closed position, when the small motor is stopped. The time required for this operation is about 3 min, during the first 15 sec of which both motors move the gate leaf 41 in. (2.7 in. per sec) and bring the movement to full speed. During the next 105 sec ($1\frac{3}{4}$ min) both motors move the gate leaf 565 in. (5.4 in. per sec) and during the final 60 sec the small motor moves the gate leaf 4 in. into its final position at an average speed of $\frac{2}{3}$ in. per sec.

To open the gates against a head, there is one additional step; namely, the operating pinion is run at low speed by the small motor for a predetermined length of time depending upon the existing difference in head, until water levels on both sides of the gate are equalized. When this has been accomplished, the large motor is automatically cut in, the movement accelerated, and the gate leaf moved into its final position in about three additional minutes, against no head. To open the gates against a 6-ft head, a period of 480 sec (8 min) in which the gate leaf travels 32 in. (0.067 in. per sec) is required for equalizing the water levels. The total time for operation against a 6-ft head thus becomes approximately 11 min. The time needed for operation against a head of 2 ft, which may be the future normal condition, is about 9 min.

The operating machinery of each gate leaf is designed to deliver a normal force of 24,000 lb to the recess side of this framed structure at the top horizontal frame and at a distance 36 ft from the vertical center line of the hinges. The maximum resistance to movement of a gate leaf is estimated at 23,700 lb, as follows:

Description	Resistance, in pounds
From current of 4 ft per sec	8,050
Movement of leaf against still water	130
Surface friction of leaf	10
Friction on pintle, from vertical load	130
Collar-bearing friction on rollers	560
Axle friction of rollers	11,100
Rolling friction of rollers	3,720
Total	23,700

Arrangements have been made for the attachment of cables to the gate leaves, near the bottom, for emergency operation. Hand-operating machinery has also been provided, with a speed reduction of 280 to 1 from the hand crank to the operating pinion. Four men turning the hand crank at a rate of 30 rpm will open or close a gate leaf in about an hour.

Operators' Houses.—An operator's house, or lock control house, has been constructed on each of the four concrete gate blocks to house the control equipment for operating the lock gates and for lighting and signaling. These houses are of reinforced concrete construction, with flat concrete roofs, or decks. The main operator's house, on the northwest gate block, is 20 ft wide, 24 ft long, and 12 ft 9 in. high, outside dimensions. The other three houses are 11 ft wide, 16 ft long, and 12 ft 9 in. high, above El. + 7.00 C.C.D. All houses have double sash windows on all four sides. Steel ladders, anchored to the walls of the houses, give outside access to the roofs, or decks, which are enclosed by pipe handrailing.

Each lock-control house contains a differential gage showing the difference in water levels between the lock and lake and between the lock and river. The main control house contains a recording gage which makes a permanent record of the water levels in the lake and in the river. Each control house is equipped with an oil-burning heater, a telephone or telephone extension, and houses a control stand and electric switchboard.

Electrical Features.—Electric energy for operation of the lock machinery, for lighting, etc., is received from the local public service company at 12,000 volts at the main lock control house (northwest operator's house), where two 12,000–480 volt power transformers and two 12,000–230–115 volt lighting transformers are used to reduce it to the proper voltage. It is carried to various outlets, for use, through conduits laid in the concrete slab, or grid, at the top of the lock.

Power lines serve a 1.5-hp and a 25-hp motor in each of the four lock control houses. Both east gate leaves can be operated simultaneously or independently from either of the easterly control houses and both west gate leaves can be operated from either of the westerly control houses. There is no interconnection between the east and the west gates.

The lighting system for the lock, guide walls, and north basin wall is a single-phase, 3-wire, 230–115 volt, system, with grounded neutral. Lighting standards (27 ft high, with a horizontal arm supporting a sodium-vapor type lamp, 23 ft above the top of the walls) are placed on each of the four gate blocks, and along the walls at 140-ft intervals.

A complete signal system has been installed, with red and green stop-and-go lights at each end of the lock. There are also red and white flashers to mark the ends of the guide walls. The flashers are gas-operated. A motor-driven air compressor with storage tank in the main control house supplies an air whistle that can be operated electrically from any one of the four lock control houses.

The telephones in the four control houses are equipped with loud gongs, and are interconnected for communication between the control houses and for outside calls at all houses.

Lock Floor.—The only part of the lock having a solid floor is that between the concrete gate blocks at the ends of the lock. The bottom of the lock proper is paved with precast reinforced concrete floor slabs, 8 in. thick. These slabs are 6 ft wide by 27 ft long. They were placed under water by means of derricks. The services of divers were required to fit the slabs closely together, this being the only operation in the construction of the entire project requiring the use of divers. The bottom of the lock was dredged and swept to proper grade and as nearly level as possible with dredging equipment. When the concrete slabs had been placed and fitted, they were leveled by placing a weight on any corner that extended above floor level and by removing the weight as soon as this part of the slab had been pressed down sufficiently into the comparatively soft clay.

The floor of precast concrete slabs is 81 ft wide and fails to cover that part of the lock bottom in the angle formed by the curved walls of the sheet pile cells. This part of the lock, back of the face of the lock walls, has no floor other than the original clay. It is not believed that this clay will scour because it is at the bottom corners of the lock where water currents set up by boat propellers will scarcely penetrate; or, if they do penetrate, they will be so diminished as to have no scouring intensity. The lock floor as originally planned was to have been formed of precast slabs 6 ft by 6 ft by 8 in. thick and the change to slabs 6 ft by 27 ft was at the request of the contractor to lessen the number of joints and labor of fitting.

Other types of lock floors were considered seriously by the designers, preferably a monolithic concrete floor placed "in the dry" and a monolithic floor placed "in the wet" by the tremie method. Any monolithic floor would have covered the entire bottom of the lock. The placing of the floor "in the dry" would have necessitated the complete dewatering of the lock; but this was not permitted although the lock walls were designed against such a contingency and it would have been an excellent test of the stability and watertightness of this type of wall. In placing the precast floor slabs, the water level inside the lock was lowered approximately 8 ft, giving a depth of water of about 16 ft in which to do this work.

A suggestion of omitting the lock floor, another of making a rubble floor of one man stone, were made but not seriously considered for the reason that propellers of large boats or tugs would create water currents at the ends of the lock which would scour the clay or move the stones possibly into positions which would interfere with the movement of the lock gates.

The floor of the lock at the ends, between the concrete gate blocks, is at a level 1 ft below the main floor and 2 ft below the top of the V-shaped curb on which the lock gates close, thus forming shallow debris pits which are expected to catch any heavy debris that might otherwise interfere with the operation of the lock gates.

Crest Wall.—A continuous reinforced concrete crest wall, 4 ft high, 2 ft 3 in. wide at the base, and 2 ft 7 in. wide at the top, with the windward or southerly face curved to throw back waves striking it, was constructed along the top of the southeast guide wall, the south lock wall, and the southwest guide wall. The northerly or lee face of this wall is vertical, paneled for architectural effect,

and located on a line 9 ft 6 in. from the lake side of the lock and guide walls. There is a clear reach of almost 1.5 miles in Chicago Harbor to the south and southeast of the lock, from which considerable waves can be produced. The northerly side of the lock is protected by the Navy Pier and an existing breakwater, both of which extend out in Chicago Harbor much farther than the lock and its guide walls.

Control Gates and Flood Gates.—Sections of wall 59 ft long in the north basin wall and in the south basin wall are constructed of concrete, supported on wooden piles, forming gate blocks in which the control gates and flood gates are installed. Four control gates and four flood gates are installed in each location. At each set of gates (control and flood) there is a passage 10 ft by 10 ft, clear opening, through the concrete wall, the top being at El. − 8.00 C.C.D., and the bottom at El. − 18.00. This should bring the top of the opening always well below the ice and debris line. On the lake side of the wall this opening is flared for a distance of 5 ft, being 12 ft wide and 13 ft high at the lake face of the wall and 10 ft square at a point 5 ft inward. The 10 ft square cross section is continued for a distance of 9 ft, where it terminates in a flume with a width of 11 ft. The bottom of the flume slopes down to El. − 19.00 C.C.D., at the inner face of the basin wall and the top slab of the basin wall, 18 in. thick, forms a roof over the flume, well above the normal water surface.

A sluice gate 10 ft $5\frac{1}{2}$ in. wide by 10 ft 3 in. high is set vertically 7 ft inside the lake face of the basin wall, or 2 ft inside the outer end of the square shaped opening (see Fig. 10). Such a sluice gate can be set at any desired opening and used to control the flow of water from the lake into the river.

At the inner end of the square shaped opening, a flap gate, 10 ft 8 in. wide and 10 ft $9\frac{1}{2}$ in. high, hinged at the top, is installed in the flume to serve as a flood gate. This flap gate is made of 8-in. by 12-in. horizontal pieces of dense Douglas fir timber, joined horizontally with 1-in. by 2-in. white oak splines and reinforced vertically with three $\frac{3}{8}$-in. by 8-in. steel plates on the lake side and by three pairs of 5-in. by $3\frac{1}{8}$-in. angles on the river side of the gate. The gate support consists of a $3\frac{1}{2}$-in. steel pipe over which rest two steel hooks attached to the upper edge of the gate. The flap gate, when in a closed position, bears against the faces of the 10-ft by 10-ft opening through the concrete gate block.

Steel plate counterweights are provided at the bottom of each flap gate, to overcome buoyancy, and thus the gate is made slightly heavier than the water it displaces. When water flows from the lake through a sluice gate, its corresponding flap gate will swing open at a vertical angle, the magnitude of which will depend upon the existing current of the water. If the flow should reverse toward the lake, the flap gate will close automatically and stop all such flow. The flap gates thus serve as automatic flood gates and are "fool proof," in that no reliance is placed upon human judgment as to when they should be closed. This feature is particularly important in this location, where the lake level frequently changes as much as 1 ft in an hour, due to a sudden shift in the direc-

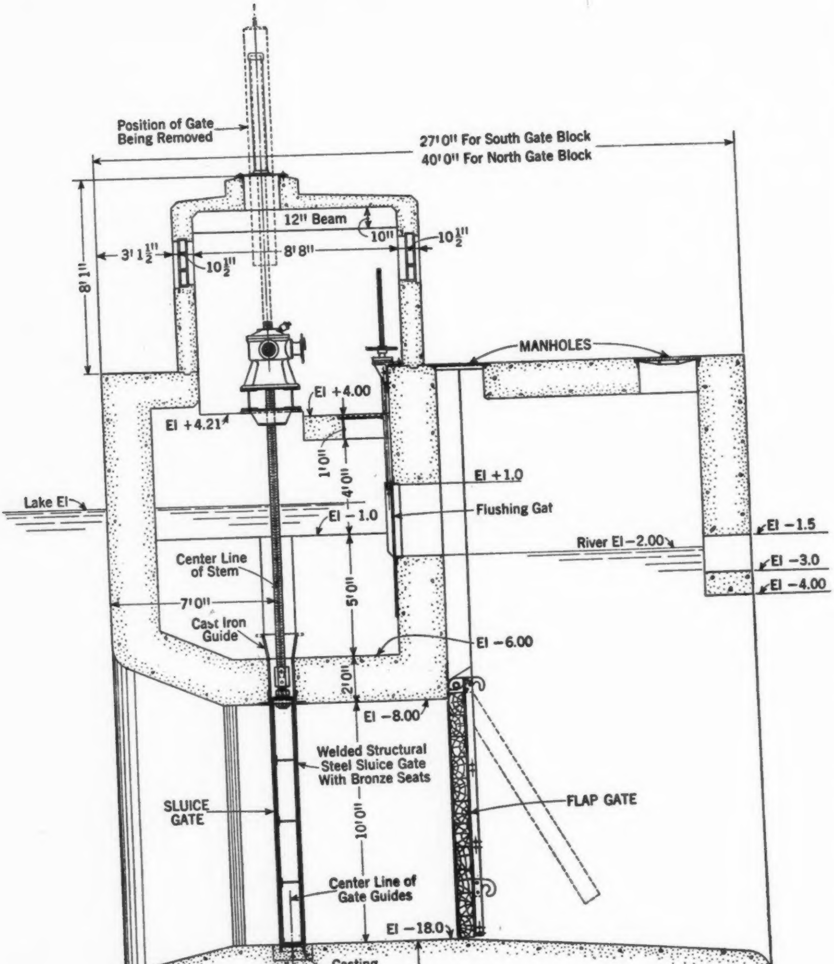


Fig. 10.—Section Through Control Gate and Flood Gate

tion of wind, and where "seiches" occasionally occur, in which lake levels fluctuate suddenly through a range of 4 to 6 ft.

The sluice gates are made of structural steel, 10-in., 20-lb channels on the outside edges, three 10-in., 25.4-lb., I-beams for the main horizontal bracing, faced with a $\frac{3}{8}$-in. steel plate. Bronze bearing strips riveted to the front and back of each gate at the sides facilitate sliding up and down in the cast-iron guide frames which are embedded in the concrete around the 10-ft square openings through the wall.

Each sluice gate has two rising stems (screws, 4 in. in diameter) which are actuated by a torque motor of approximately 5 hp and 3,600 rpm, operating through a proper set of speed-reducing gears down to a speed of 24 rpm at the screw stem. The gate can be lifted at the rate of 12 in. per min. Hand operation is also provided. The operating mechanism is mounted on the floor of an operating gallery at El. + 4.00 C.C.D., and the motor is 2 ft higher and is therefore always above the water line.

Two operating galleries, each 10 ft 5 in. wide, 70 ft long, and 8 ft high, above the basin walls, house the operating mechanism. The construction of these galleries is similar to that of the control houses on the lock, reinforced concrete sides and roof. Glass brick is used for windows. One of these galleries is located on the north basin wall and the other on the south basin wall.

Schedule of Construction

The order of construction was first, lock and guide walls, then south basin wall, and finally, north basin wall. A small contract for driving test piles was awarded May 21, 1936, and the work was performed within two weeks. The information thus obtained checked assumptions previously made and used in the design of lock and walls.

A contract for the construction of the lock and guide walls, the longest job, was awarded July 9, 1936. The contract for electrical work was awarded November 16, 1936, and the contract for the north and south basin walls and control gates March 16, 1938.

The construction of the lock and guide walls required two months more than two construction seasons and the lock was formally dedicated and opened for traffic September 7, 1938. The basin walls and control gates were constructed during the season of 1938 except for the concrete cap on the north basin wall which was finished in June, 1939.

Since navigation into the Chicago River could not be interrupted, the construction of the north basin wall had to be delayed until the lock could be opened for traffic, or until September, 1938. When the reduction in diversion of water from the Lake Michigan watershed to 1,500 cu ft per sec, annual average, was made December 31, 1938, the Chicago River Control Works were ready to function. The work remaining to be done on the north basin wall was all above the water line.

Costs

The approximate cost of the Chicago River Control Works was $2,704,000, distributed among the various items, as follows:

Test piles		$ 4,600
Lock:		
Cofferdams	$105,000	
Gate structures	554,000	
Gates	177,000	
Walls	467,000	
Floor	42,000	
Control houses	23,000	
Fender piles	5,000	
		1,373,000
Guide walls		579,000
South basin wall		223,000
North basin wall		53,000
Control Works:		
Gate blocks	114,000	
Gates	32,000	
Operating galleries	6,000	
		152,000
Electrical work		73,700
Miscellaneous work		14,100
Field office		7,000
		2,479,400
Government supervision (1.05%)		26,000
Engineering expense—design, supervision, inspection, testing (8.01%)		198,600
Total cost		$2,704,000

Operation of Lock

In operating the lock gates the first step is to equalize the water surfaces on the two sides of the gate by separating the gate leaves slowly and letting the water flow between them as well as through the two openings at the sides of the lock between the faces of each gate leaf and the edge of the concrete wall of the gate recess. With the operating machinery as now installed, the gate leaves separate at the center at the rate of 4 in. per min. The combined openings at the sides of the lock are approximately 15% greater than the center opening; that is, when the outer edge of a gate leaf has traveled 6 in. from the

center line of the lock, its other edge has moved back 6 in. into the gate recess and has left an opening about 7 in. wide between the concrete edge of this recess and the face plate of the gate. When the gate leaves are separated 12 in. at the center of the lock, the combined width of the side openings is about 14 in. Essential flow areas, clearances, and data are shown in Figs. 11 and 12.

Formulas have been developed for calculating the time required to fill or empty the lock for a given difference in head. The formulas have been based on the assumption that the total widening of the three openings past a set of gates is at a uniform rate of 9 in. per min. The formula used for determining the dis-

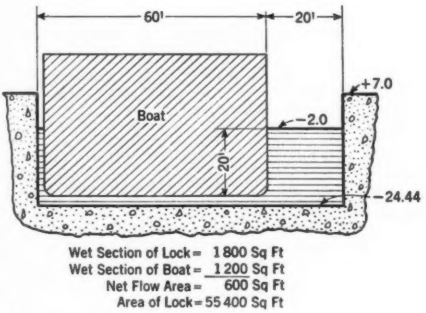


FIG. 11.—TYPICAL CROSS SECTION OF LOCK

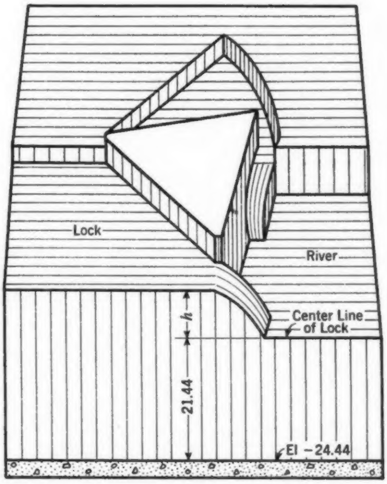


FIG. 12.—LONGITUDINAL CROSS SECTION OF LOCK

charge, in cubic feet per second, for any given condition is a combination of the Francis weir formula,

$$Q = 3.33\ b\ h^{1.5} \qquad (1)$$

and the formula for discharge through a submerged orifice,

$$Q = A\ (2\ g\ h)^{0.5} \qquad (2)$$

to both of which are applied a contraction coefficient of 0.95.

The formula for rate of filling the lock is:

$$Q = 7.62\ b\ d\ h^{0.5} - 4.45\ b\ h^{1.5} \qquad (3a)$$

The formula for emptying the lock is:

$$Q = 7.62\ b\ d\ h^{0.5} + 3.17\ b\ h^{1.5} \qquad (3b)$$

in which Q = discharge, in cubic feet per second; b = total width, in feet, of

three orifices; d = depth of water, in feet, in orifices at end of filling or emptying operation; and h = head, in feet, between water inside lock and water outside lock, at any instant.

The corresponding formulas for time, in seconds, are as follows—for filling lock:

$$T = \frac{1{,}872}{d^{0.25}} \left[\log \frac{1.31\, d^{0.5} + h_1^{0.5}}{1.31\, d^{0.5} - h_1^{0.5}} - \log \frac{1.31\, d^{0.5} + h^{0.5}}{1.31\, d^{0.5} - h^{0.5}} \right]^{0.5} \ldots\ldots (4a)$$

for emptying the lock:

$$T = \frac{1{,}900}{d^{0.25}} \left[\tan^{-1} \left(\frac{h_1}{2.40\, d} \right)^{0.5} - \tan^{-1} \left(\frac{h}{2.40\, d} \right)^{0.5} \right]^{0.5} \ldots\ldots (4b)$$

in which h_1 is the head, in feet, when gates start opening (a constant).

A diagram giving all the data in connection with filling the lock from the lake at water level + 3.00 has been developed and is shown in Table 1(a) and

TABLE 1.—Discharge Quantities for a Rise or Fall of 6 Ft (See Fig. 13)

Linear movement of operating struts, in inches[a]	Cumulative time, in seconds[b]	(a) A 6-Ft Fill			(b) A 6-Ft Drop		
		Average rate of flow, in each 30 sec[c]	Discharge in 30 sec, in cubic feet	Total discharge, in cubic feet	Average rate of flow, in each 30 sec[c]	Discharge in 30 sec, in cubic feet	Total discharge, in cubic feet
(1)	(2)	(3)	(4)	(5)	(3)	(4)	(5)
1.2	30	90	2,700	2,700	90	2,700	2,700
2.4	60	240	7,200	9,900	250	7,500	10,200
3.6	90	410	12,300	22,200	410	12,300	22,500
4.8	120	570	17,100	39,300	550	16,500	39,000
6.0	150	710	21,300	60,600	680	20,400	59,400
7.2	180	850	25,500	86,100	780	23,400	82,800
8.4	210	960	28,800	114,900	880	26,400	109,200
9.6	240	1,040	31,200	146,100	940	28,200	137,400
10.8	270	1,100	33,000	179,100	990	29,700	167,100
12.0	300	1,120	33,600	212,700	1,010	30,300	197,400
13.2	330	1,080	32,400	245,100	1,000	30,000	227,400
14.4	360	1,000	30,000	275,100	930	27,900	255,300
15.6	390	850	25,500	300,600	850	25,500	280,800
16.8	420	600	18,000	318,600	710	21,300	302,100
18.0	450	300	9,000	327,600	560	16,800	318,900
19.2	480	....	....	....	400	12,000	330,900
20.4	510	....	....	....	40	1,200	332,100

[a] Or of the pitch circle (diameter, 20.37 in.) on the main pinion. [b] Time influences these data on the basis of the orifice on the center line of the lock widening at the rate of 4 in. per min, with the main pinion operating at 0.0375 rpm. [c] In cubic feet per second.

Fig. 13(a). The river and the water in the lock are h feet below the lake. For any head differential, these data show the rate of inflow into the lock, the variation in head, the increasing width of opening, and the time. The formula for flow into the lock under this head condition is

$$Q = 0.95\, b\, [(27.44 - h)\, (2\, g\, h)^{0.5} + 3.33\, h^{1.5}] \ldots\ldots\ldots\ldots (5)$$

$\frac{Q}{b}$	$210h^{0.5}$	$4.45h^{1.5}$	$h^{1.5}$	$h^{0.5}$	Elevation of Water Surface in Lock, in Feet	Distance h, in Feet, Above Lower Water Surface
450	515	65	14.7	2.45	−3.00	6.0
434	492	58	12.9	2.34		5.5
420	470	50	11.2	2.24	−2.00	5.0
402	445	43	9.6	2.12		4.5
384	420	36	8.0	2.00	−1.00	4.0
364	393	29	6.5	1.87		3.5
340	363	23	5.2	1.73	0.00	3.0
314	332	18	4.0	1.58		2.5
283	296	13	2.8	1.41	+1.00	2.0
248	256	8	1.8	1.22		1.5
205	210	5	1.0	1.0	+2.00	1.0
148	149	1	0.35	0.71	+2.50	0.5
0	0	0	0	0	+3.00	0
$\frac{Q}{b}$	$3.16h^{1.5}$	$163h^{0.5}$	$h^{1.5}$	$h^{0.5}$		
446	46.5	400	14.7	2.45	+3.00	6.0
422	40.8	381	12.9	2.34		5.5
400	35.4	365	11.2	2.24	+2.00	5.0
375	30.3	345	9.6	2.12		4.5
351	25.3	326	8.0	2.00	+1.00	4.0
326	20.5	305	6.5	1.87		3.5
300	16.4	283	5.2	1.73	0.00	3.0
271	12.6	258	4.0	1.58		2.5
240	8.93	231	2.8	1.41	−1.00	2.0
205	5.70	199	1.8	1.22		1.5
166	3.16	163	1.0	1.0	−2.00	1.0
116	1.12	115	0.35	0.71	−2.50	0.5
0	0	0	0	0	−3.00	0

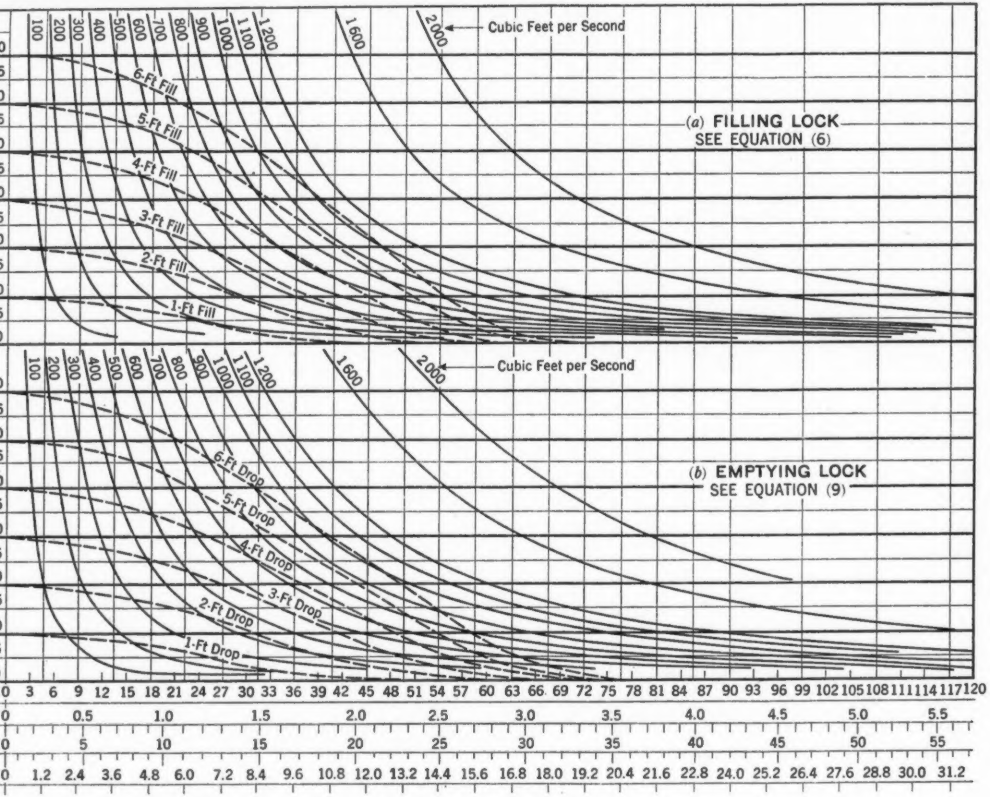


FIG. 13.—FILLING AND EMPTYING DATA; CHICAGO RIVER CONTROL WORKS

in which: h = head differential, in feet; 0.95 = orifice coefficient; 27.44 = depth of lake, in feet, above the bottom of the lock; and b = total width, in feet, of orifice.

Simplified, Eq. 5 is

$$Q = 210\, b\, h^{0.5} - 4.45\, b\, h^{1.5} \qquad (6)$$

The derivation of the equation for filling the lock from the lake at El. + 3.00 with the total of the three openings of the gates widening at a constant rate of 9 in. per min is as follows: $dD = \frac{Q}{A}\, dt = \left(\frac{210\, b\, h^{0.5} - 4.45\, b\, h^{1.5}}{55{,}400}\right) dt$ $= \left[\frac{210\,(h_1 - D)^{0.5} - 4.45\,(h_1 - D)^{1.5}}{55{,}400} \times \frac{t}{80}\right] dt$, or:

$$t\, dt = \left[\frac{996{,}000}{47.2\,(h_1 - D)^{0.5} - (h_1 - D)^{1.5}}\right] dD \qquad (7a)$$

in which: h_1 = head in feet when gates start opening (a constant); h = head in feet at any time during opening; D = total drop in head (in feet) at any time during opening $= h_1 - h$; t = time in seconds for head to drop from h_1 to h; Q = discharge in cubic feet per second $= 210\, b\, h^{0.5} - 4.45\, b\, h^{1.5}$; b = total width of openings in feet $= t \div 80$; and A = area of lock surface = 55,400 sq ft. Let $u = h_1 - D$. Then $du = -\,dD$; $dD = -\,du$; and

$$t\, dt = \left(\frac{-\,996{,}000}{47.2\, u^{0.5} - u^{1.5}}\right) du \qquad (7b)$$

Let $v = u^{0.5} = (h_1 - D)^{0.5}$. Then $dv = \frac{1}{2}\, u^{-0.5}\, du$; $du = 2\, u^{0.5}$; $dv = 2\, v\, dv$; and

$$t\, dt = \left(\frac{-\,996{,}000 \times 2\, v}{47.2\, v - v^3}\right) dv = \left(\frac{-\,1{,}992{,}000}{47.2 - v^2}\right) dv \qquad (7c)$$

Integrating and simplifying: $t^2 = \left[-\,672{,}000 \times \log \frac{6.86 + (h_1 - D)^{0.5}}{6.86 - (h_1 - D)^{0.5}}\right] + C.$

When $D = 0$, $t^2 = 0$. Then, $C = +\,672{,}000 \times \log \frac{6.86 + h_1^{0.5}}{6.86 - h_1^{0.5}}$; and t^2 $= 672{,}000 \left[\log \frac{6.86 + h_1^{0.5}}{6.86 - h_1^{0.5}} - \log \frac{6.86 + (h_1 - D)^{0.5}}{6.86 - (h_1 - D)^{0.5}}\right]$; but $h_1 - D = h$; and, therefore,

$$t = 820 \log \left[\frac{6.86 + h_1^{0.5}}{6.86 - h_1^{0.5}} - \log \frac{6.86 + h^{0.5}}{6.86 - h^{0.5}}\right]^{0.5} \qquad (8)$$

The data for the filling diagram were developed with the aid of Eq. 6 for flow and Eq. 8 for time of filling.

Table 1(b) and Fig. 13(b) contain the data in connection with emptying the lock into the river at water level − 3.00 (see Fig. 12). The formula for flow

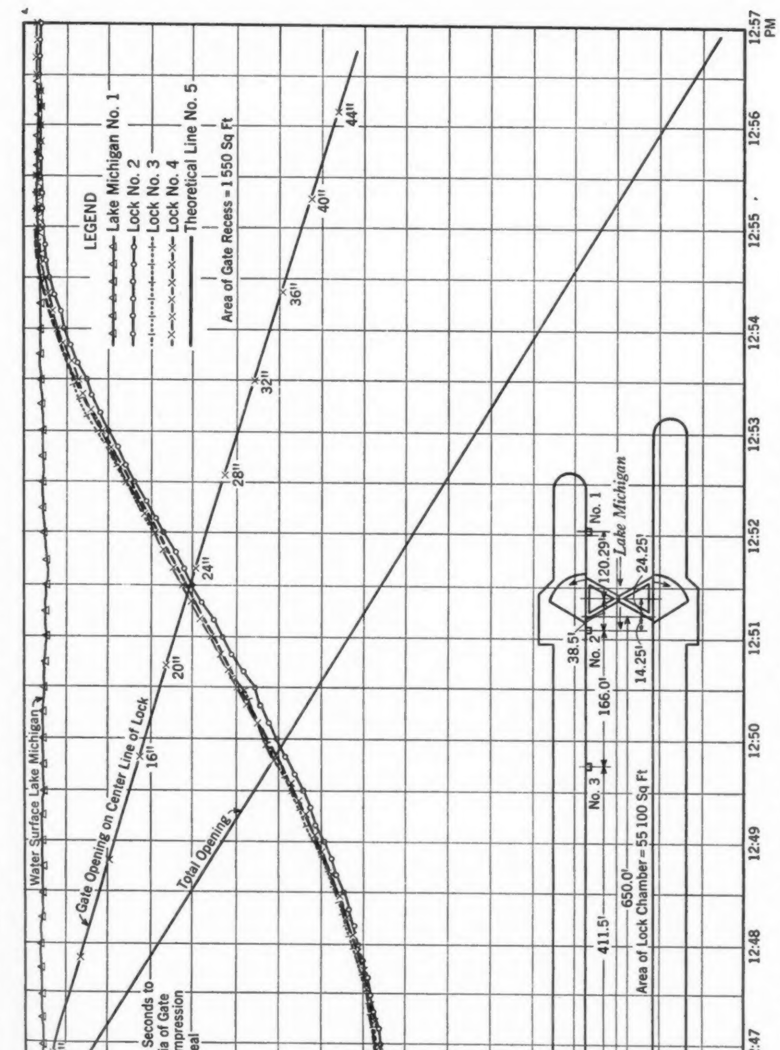


FIG. 14.—TIME REQUIRED TO FILL LOCK CHAMBER, OPERATING GATES AT SLOW SPEED; JUNE 25, 1938

from the lock under this head condition is

$$Q = 0.95\, b\, [21.44\, (2\, g\, h)^{0.5} + 3.33\, h^{1.5}] = 163\, b\, h^{0.5} + 3.17\, b\, h^{1.5} \quad \ldots (9)$$

The derivation of the equation for emptying the lock into the river at El. − 3.00 with the total of the three openings of the gates widening at a constant rate of 9 in. per min is as follows:

$$dD = \frac{Q}{A}\, dt = \left(\frac{163\, b\, h^{0.5} + 3.17\, b\, h^{1.5}}{55{,}400} \right) dt \quad \ldots\ldots\ldots\ldots (10)$$

Following the same procedure as in Eq. 7,

$$t = 885 \left[\tan^{-1} \left(\frac{h_1}{51.4} \right)^{0.5} - \tan^{-1} \left(\frac{h}{51.4} \right)^{0.5} \right]^{0.5} \quad \ldots\ldots\ldots\ldots (11)$$

The data for Fig. 13(*b*) were developed with the aid of Eq. 10 for flow and Eq. 11 for time of filling. The formulas for filling and emptying the lock under the conditions of lake level at + 3.00, and river level at − 3.00, and the curves in Fig. 13 were prepared as a basis for study of the design and operation of the lock. However, for the normal range of lake levels to be expected, and for all practical purposes, the filling times for various starting head differentials can be read from these curves with very slight percentage of error.

An actual test of time of filling the lock was made June 5, 1938, with the water level at El. − 0.50 C.C.D. outside the lock and at El. − 8.60 inside the lock at the beginning of the test. The results of this test checked within 1% of the theoretical results computed from the formula for time of filling the lock. Fig. 14 contains curves of the water levels at three points inside the lock and of the lake outside the lock during the test. The heavy dash line is the theoretical line, plotted from data derived from the general formulas for flow and for time of filling.

In equalizing the water surfaces on the two sides of the gates by permitting the water to flow through orifices between and around the gates there are always two variables. The combined width of the three openings is increasing at a practically constant rate, the head is lessening at a varying rate and the depth of the openings is lessening at a rate similar to the head, but of comparatively small effect considering the total depth. The net result is a decreasing head and an increasing area of opening. The flow at first is at maximum velocity because of maximum head, but of small volume because of minimum area. The volume of flow increases to its maximum when about 60% of the width of the flow opening has been attained, the head then being about 40% of the maximum or initial head. The volume of flow then decreases (more sharply than it had increased) until near the end of the time of equalization the area of the orifices is maximum and the velocity is minimum.

It is important that the volume of inflow to the lock should be kept low in order that no currents should be set up inside the lock which might be disturbing to water craft in the lock at the time. In the test of June, 1938, with a maximum initial head of 8 ft—the worst possible condition—the maximum rate of

inflow was about 1,300 cu ft per sec. Approximately 600 cu ft per sec of this came through the central opening and about 350 cu ft per sec from each of the side openings. The velocity of water longitudinally down the lock at the influent end could not exceed 1.5 ft per sec, with a flow of 1,300 cu ft per sec and a cross section of 2,000 sq ft. At the opposite end, of course, the current was zero. Even with a large boat in the lock, reducing its cross section, no current could be created dangerous to craft inside the lock.

Benefits from Control Works

The only disadvantage caused by the control works at the mouth of the Chicago River will apply to navigation. These works necessitate the locking of every boat entering the Chicago River from Lake Michigan or passing from the river to the lake, entailing a delay of approximately 15 min for each operation. This penalizes only a small percentage of the lake boats using Chicago Harbor since most of the larger boats dock at the Navy Pier outside the control works. The facilities at this pier are ample.

The advantages of the control works are the creation of an inner harbor, the better control over flow in the Chicago River and Main Drainage Canal and, most important, the insurance against the reversal of flow of the Chicago River into Lake Michigan.

The inner harbor, about 2,000 ft long from north to south and about 1,700 ft wide from east to west, comprises about 17 acres of protected water, safe for river barges. This harbor is now used for the interchange of cargoes between lake boats and river barges and a future development will undoubtedly be the interchange of such freight across the north pier, widened and provided with warehouses and unloading and conveying equipment.

The regulation of flow in the river and canal system in the past has been solely by the manipulation of the controlling works at Lockport, 36 miles inland from the mouth of the Chicago River. Close control over flow in the Chicago River has been difficult, at times, because of sudden fluctuations of Lake Michigan and because of inflow from rainfall runoff which could occur in the Chicago River in less time than that required to change flow conditions by operating the Lockport Controlling Works. The level of Lake Michigan frequently changes as much as 1 ft in one hour due to a shift in wind. Runoff from rains can enter the Chicago River in volume within three hours after rain has started to fall. It can enter the Main Chicago River within less than one hour, from the "loop" area. Three hours will elapse after a change in flow conditions is made at Lockport before the first effect of this change can be noticed in the Main Chicago River and from six to nine hours are required to produce a substantial change in flow conditions near the mouth of this river. For this reason, in the past, it was necessary to anticipate rainfall by three to six hours and establish flow conditions which would permit the inflow from storms without danger of reversal of the river.

When the average flow in the canal was around 10,000 cu ft per sec it was necessary to increase the flow only two or three times per year to handle the worst storms. The maximum runoff from rains is now about 12,000 cu ft per

sec. This will increase in the future, as paved areas in Chicago increase. When the average flow in the canal was around 6,600 cu ft per sec (diversion from Lake Michigan watershed 5,000 cu ft per sec, annual average, in addition to 1,600 cu ft per sec sewage flow) it was necessary to increase the flow 15 to 20 times per year to handle runoff from the ten to twelve major storms which occur in a year. Flow conditions, of course, were established in anticipation of some rains which did not occur and for some rains which produced less volume of runoff than was expected. With the present average flow in the canal of 3,200 cu ft per sec (1,700 cu ft per sec sewage or domestic pumpage and 1,500 cu ft per sec diversion from Lake Michigan watershed) it would be necessary to increase the flow sixty to ninety times per year if it were not for the Chicago River Control Works. The number of rains in a year in the Chicago area average about 55 at an individual rain-gaging station and about 90 over a composite of all the observation stations in the district.

As previously stated, of the present total average flow of 3,200 cu ft per sec, about 500 cu ft per sec, annual average, is used for flood runoff from the Chicago and Calumet River watersheds, making the average low water flow that will prevail two thirds of the time approximately 2,700 cu ft per sec. This will require a drop of only 6 in. between Lake Michigan and Lockport which will produce a very slight slope indeed (almost flat) and a maximum velocity of 0.7 ft per sec, with less than 0.2 ft per sec in the main Chicago River. This 2,700 cu ft per sec will be composed of 1,700 cu ft per sec of domestic pumpage (sewage or effluent from sewage treatment plants), and of 1,000 cu ft per sec of fresh water diverted from Lake Michigan. This water from Lake Michigan is that deemed "necessary for the purpose of maintaining navigation in the Chicago River, as a part of the Port of Chicago." Of course, a bad sanitary condition, approaching a nuisance, will be produced in the upper reaches of the Lakes-to-Gulf Waterway, with a sluggish current, and with only 1,000 cu ft per sec of fresh water available to dilute the effluent from sewage treatment plants through which have passed the sewage from an equivalent population of 6,500,000. Although this sewage may be treated to an extent comparable to 85% purification, the residue will be practically equal in effect to the raw sewage from 975,000 people, or 15% of 6,500,000.

With the operation of the Chicago River Control Works in conjunction with the manipulation of the Lockport Controlling Works it will be possible to produce a change in flow conditions in the canal and river system in much less time (perhaps two thirds of the time) than that required with control only at the downstream end of the system. The effect of fluctuations of Lake Michigan water levels on the flow in the Chicago River will be eliminated. The water surface in the Chicago River can be held at any desirable predetermined level and definite vertical underclearance of bridges across the river maintained, except during the infrequent times of flood inflow to the river. If the river level is held low to maintain bridge underclearance, in the interest of navigation, there will be additional volume in the river channel to receive runoff from rains, and this may be sufficient to absorb the additional flow produced by many minor rains without any modification of flow conditions. The inflow

from heavy storms will necessitate increasing the flow throughout the entire channel. The water surface slope for this condition can be produced more quickly in the future than in the past by lowering the water surface at Lockport while the river level at Chicago is rising from flood inflow. The vertical clearance under bridges across the Chicago River would be lessened temporarily but even this condition can be corrected in a comparatively short time.

The most important advantage from the Chicago River Control Works, and that which may be termed the primary purpose of such works, is the protection that will be afforded against reversal of the Chicago River into Lake Michigan under any condition. Such reversal will be prevented by the automatic closing of the flood gates in case the river level rises higher than that of the lake. Thus the protection of the purity of the water supply of the Chicago region from contamination by sewage is assured, as far as is possible, along the entire shore line of the State of Illinois; and this has been the primary function of The Sanitary District of Chicago for the past half century.

Acknowledgments

The Chicago River Control Works were constructed by The Sanitary District of Chicago as a project of the Public Works Administration (PWA) and as a part of the sewage treatment construction program made necessary by the decree of the U. S. Supreme Court on April 21, 1930. Acknowledgment is made to Ross A. Woodhull, President of The Sanitary District and William H. Trinkaus, M. Am. Soc. C. E., chief engineer. The project was designed under the direction of the writer by Ralph R. Leffler, engineer of structural design, and constructed under direction of L. B. Barker, M. Am. Soc. C. E., engineer of construction, and Harry J. Bartz, assistant civil engineer. Joshua D'Esposito, M. Am. Soc. C. E., was resident projects engineer, PWA, and U. F. Turpin, M. Am. Soc. C. E., assistant resident projects engineer, PWA. Major Karrick was the U. S. District Engineer at Chicago. The lock and guide walls were constructed by the Frazier-Davis Construction Company, and G. L. Tarleton, Contractor, Inc.; the basin walls and control gates by the Ketler-Elliott Company; and the electrical work was done by the A. S. Schulman Electric Company.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

ENGINEERING GEOLOGY PROBLEMS AT CONCHAS DAM, NEW MEXICO

Discussion

BY IRVING B. CROSBY, AFFILIATE, AM. SOC. C. E.

IRVING B. CROSBY,[10] AFFILIATE, AM. SOC. C. E. (by letter).[10a]—Mr. Johnson has presented some additional data on the geological and foundation problems at Conchas Dam. He states that the conclusion that the percentage of voids was higher in the south abutment than in the north abutment was based upon initial and incomplete tests and that the complete data later available indicate that the physical characteristics of the shale in the south and north abutments are practically identical. However, the original tests cited in the paper were all that were available after investigation of the foundations had been under way for more than two years.

The writer stated, as information from official sources, that the average shearing strength of the red shale was determined from compression tests on twenty 2-in. cubes; but Mr. Johnson states that all shear values entering into the design and stability computations were derived from direct shear tests on cylinders of shale 6 in. in diameter. Since it is known that direct shear tests on shale give less reliable results than shearing values calculated from compression tests, the writer did not suppose that the results of the compression tests would be discarded at a late date in favor of results obtained by a less reliable method. The most accurate method of determining the shearing resistance of this shale would be by the triaxial compression tests.

In regard to possible settlement of that part of the dam underlain by red shale, the writer stated that consolidation tests had indicated that the maximum settlement under extreme conditions would be about 4 in. after a period of years. Mr. Johnson states: "However, it is doubtful that the settlement differential will approach the 4 in. indicated by the author, whose computation is apparently based on the omission of the lower red shale from consideration or on the assumption that the lower red shale will be equally reconsolidated under the entire dam." The writer did not omit the lower red shale from consideration, but the problem is extremely complicated. Certain facts essential for a rigorous solution were not obtainable, and simplification of the problem was necessary.

The formations at the dam site were laid down as flat beds in water and were

NOTE.—This paper by Irving B. Crosby, affiliate, Am. Soc. C. E., was published in January, 1939, *Proceedings*. Discussion on this paper has appeared in *Proceedings* as follows: April, 1939, by H. L. Johnson, Esq.

[10] Cons. Eng. Geologist, Boston, Mass.

[10a] Received by the Secretary December 9, 1939.

buried by succeeding formations until there were at least 1 250 ft of sediments on top of the upper red shale. The lower red shale was buried approximately 150 ft deeper and had 1 400 ft of sediment on top of it. Therefore, other things being equal, the compaction of the two shales should vary as the loads upon them. About 1 225 ft of overlying rocks have been removed from above the upper red shale on the north abutment, reducing the load by 98% and allowing expansion of the shale. The dam on the north abutment will restore approximately 8% of this load and will cause some recompaction of the shale. In the canyon, erosion cut a maximum of 140 ft deeper, leaving only 70 ft of rock on the lower red shale and reducing the thickness of overlying rocks by 95 per cent. The highest part of the dam would restore about 18% of the original load. However, the lower red shale was unloaded only over a span of 300 ft, and it was covered by approximately 70 ft of strong artesian sandstone which would have some restraining effect. If the underlying shale had expanded appreciably, it should have uplifted the sandstone, with a tendency to open cracks in it. The fact that the sandstone contains water under artesian pressure indicates that the cracks have not been opened appreciably. Therefore, the amount of expansion and, consequently, the amount of possible recompaction of the lower red shale is uncertain. Since this shale is less than half as thick as the upper red shale, the possible settlement due to compaction of the lower red shale is less. Since undisturbed samples of the lower red shale were not available and, therefore, since it was impossible to estimate the compaction of this shale under a given load from a consolidation test, it was not possible to evaluate the settlement which might be caused by placing the dam on overlying rocks.

It was desired, however, to estimate the effects of the worst possible conditions and, therefore, an estimate was made from consolidation tests of maximum possible settlement due to recompaction of the upper red shale from the load of the dam. The possible equalizing effect of differential recompaction of the lower red shale was disregarded purposely since it is uncertain and cannot be evaluated. It was realized that the estimates made gave an extreme value, probably considerably greater than could occur, as was stated in the writer's report, but that if the dam could be designed to meet this extreme condition it would be safe in any case. Since the dam could be designed to meet these extreme conditions without difficulty, this was done. An attempt to evaluate the uncertain effects of the lower red shale would not have made the dam any safer or have resulted in any important economy.

Mr. Johnson discusses the compressibility and elasticity of artesian aquifers without mentioning that these had been covered in the paper and that the conclusion had been reached that such movement would be uniform and harmless.

The special joint with sealing wells between the abutments and abutting monoliths is described in some detail by Mr. Johnson who thus supplements the paper on that subject referred to by the writer.[5] Mr. Johnson also gives additional information about the grouting program which was completed subsequent to the presentation of the paper. He also describes the most recent provisions made to study foundation settlement, and thus brings up to date (1939) the published information on Conchas Dam.

[5] For description of joint, see "Dam Building on Difficult Rock," *Engineering News-Record*, June 9 1938, pp. 808–809.

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DISCUSSIONS

LATERAL SPILLWAY CHANNELS

Discussion

BY THOMAS R. CAMP, M. AM. SOC. C. E.

THOMAS R. CAMP,[16] M. AM. SOC. C. E. (by letter).[16a]—The discussion of this paper has been gratifying particularly because of its quality. Professor Howland presents a helpful rule for approximating the drop in water surface quickly. Professor Carpenter and Messrs. Vennard and Whitley suggest a unique procedure for simplifying design, whereby both the slope and friction terms in the equation are eliminated by equating them. Mr. Knapp rearranges the differential equation in a manner to indicate each factor which contributes to the draw-down of the water surface, and he also points out the applicability of the differential equation to side weirs. Mr. Rohwer rightfully questions the utility of the Hinds-Favre equation for large spillways where violent rollers and entrained air are of paramount importance. Mr. Thomas presents a useful development of the economics of wash-water gutter design. The writer wishes to express his appreciation for these helpful contributions.

Professor Howland's Equation (41) for computing the drop in water surface between two sections a distance Δx apart is of general applicability, regardless of the shape of the flume. To this extent it is more useful than the writer's integrated form (Equation (23)), which applies only to rectangular channels of constant width and slope. The use of Equation (41) for the more simple channels, however, is more laborious than the writer's method. Even for the evaluation of f from measurements of the water surface, Equation (23) is probably more direct and more simple than Equation (41). It possesses the added advantage of producing, immediately, the average value of f for the entire stretch of channel being investigated.

In a first approximation of the value of H_0 in channels where the depth d at a section at or near the lower end is known, Professor Howland neglects the second and third terms of Equation (41). The total drop is thus approximately

NOTE.—This paper by Thomas R. Camp, M. Am. Soc. C. E., was published in February, 1939, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: May, 1939, by Messrs. W. E. Howland, Lewis V. Carpenter and John K. Vennard and Fenner H. Whitley, F. Knapp, and Carl Rohwer, and June, 1939, by Harold Allen Thomas, Jr., Esq.

[16] Associate Prof., San. Eng., Mass. Inst. Tech., Cambridge, Mass.

[16a] Received by the Secretary November 22, 1939.

equal to twice the velocity head at the section where d is known. This approximation is not a safe procedure, although in the examples cited it does produce results that are not widely in error. The last two terms in Equation (41) do tend to compensate for one another, but the assumption that they are equal may be too greatly in error for reliability. The second term depends upon the slope of the bottom, is negative for a level bottom, approaches zero as the bottom slope becomes nearly parallel with the water surface, and may become positive for bottoms with steep slopes. For a level bottom and a hydrostatic control section at the lower end, the sum of the first two terms is $1.46 \frac{V^2}{2g}$, as may be seen from Equation (45). The drop in the water surface due to the friction term bears little relation to the second term of Equation (41). In the writer's examples the friction drop is from 6.5% to 16% of the total drop.

Professor Howland's proposal to use vanes in order to utilize a part of the momentum of the inflowing water and thus increase the capacity of the channel is worthy of investigation for dam spillways. Some knowledge of the value of the friction losses in such large channels is necessary, however, before one can be sure that the vanes are worth their cost. Undoubtedly, the friction losses are large, but it is doubtful if they are so large as to permit of an increase in capacity of 50% by their reduction.

Messrs. Carpenter, Vennard, and Whitley note that the values of f as found by the writer for lateral spillway channels are greater than the usual values of 0.02 or 0.025 for uniform flow in pipes. The increase in the value of f is undoubtedly due to the greater turbulence resulting from the impact of the incoming water. The table of values of f as presented by the writer is indeed meager, but all data which were available were used. It is hoped that other data will become available in the future for the evaluation of f. It will be very helpful if values of f can be obtained from some full-scale dam spillways where air entrainment and rollers are pronounced. Until some such efforts are made, the validity of the Hinds equation for the design of such structures is in doubt. It will be noted from the data presented in Table 1 that the value of f increases as the increment of inflow increases. One may surmise that with dam spillway channels, where the inflow increment is much greater, the value of f may be very much larger than the values in Table 1.

The writer's statement (see heading "Critical Depth") that "For the design of freely discharging flumes, a close approximation to actual conditions will be obtained if a hydrostatic control section is assumed to exist at a distance up stream from the end equal to three or four times the critical depth" has been criticized by Messrs. Carpenter, Vennard, and Whitley and again by Mr. Thomas. The proper distance to take, of course, will be influenced by the shape of the outlet end of the flume, the discharge, the amount of submergence or degree of aeration of the discharging stream, and probably other factors. If a hydrostatic control section is assumed to exist at the end, the design will be on the safe side, but it may be uneconomical. The arbitrary figure of "three or four" times the critical depth should be tempered by the judgment of the designer. In the case of the Detroit experiments (Fig. 4), the distance was

3.8 and 4.3 times the critical depth for Run No. 1 and Run No. 2, respectively. In the case of the Baldwin Filtration Plant experiment (Fig. 5(*b*)), the distance was 1.25 times the critical depth. For the Division Avenue Plant experiment (Fig. 5(*a*)) the critical depth was substantially at the end of the channel; but this channel can scarcely be said to be discharging freely. The experimental channel at the Massachusetts Institute of Technology was provided with means below the outlet end for controlling the depth of flow, and for the run presented in Table 1 and Fig. 6 the outlet end was submerged.

In applying the arbitrary rule of three or four times the critical depth, the value of d_c should not be computed from the total channel discharge, but rather from the inflow up stream from the assumed position of the hydrostatic control section. As has been noted by Messrs. Carpenter, Vennard, and Whitley, a portion of the wash water will enter the channel below the control. Since this water is discharged at lower stage, it does not influence the shape of the water-surface curve above the control.

If a hydrostatic control is assumed to exist at the outlet end, the computed value of H_0 will be too great. If a hydrostatic control is assumed at the proper distance up stream but the total discharge is used for computing d_c and H_0, the value of H_0 will be too great again. Moreover, since q has thus been made too great, the water-surface curve will be distorted. If the hydrostatic control is assumed at the proper distance up stream and computations are based upon the inflow above this section, a correct value of H_0 will result, and the computed shape of the water surface curve will coincide with the actual almost down to the assumed position of the control. It will be noted from Figs. 4 and 5, however, that the actual depth at the assumed position of the control is likely to be greater than d_c. This is due to the prevalence of vertical accelerations in this region which result in pressures less than hydrostatic.

The approximate values of $\bar{d}$ and $\bar{R}$ as given in Equations (47*a*) and (47*b*) are based upon a straight water-surface profile. Since the profile is more nearly parabolic in shape, better approximations for $\bar{d}$ and $\bar{R}$ may be obtained from the following equations:

$$\bar{d} = \frac{2\,H_0 + d_c}{3} \qquad (64a)$$

and

$$\bar{R} = \frac{\left(\dfrac{2\,H_0 + d_c}{3}\right) b}{2\left(\dfrac{2\,H_0 + d_c}{3}\right) + b} \qquad (64b)$$

If design data are computed for the two 20-ft wash-water troughs of Table 4 on the basis of these relations and Equation (46), the ratio $\frac{S}{f}$ becomes 0.052 for the trough 1 ft wide and 0.025 for the 2-ft trough.

The scheme proposed by Messrs. Carpenter, Vennard, and Whitley for simplifying the design of wash-water gutters by equating the slope term to the friction term, as shown by Equation (44), raises the question of the desirability

of fixing the slope for the sole purpose of simplifying computations. If this is the aim, a simpler procedure is to make the bottom level. With a level bottom, H_0 may be approximated very quickly by computing the drop in the water surface from Equation (45) and increasing it by about 10% to allow for friction. There are advantages in a level bottom from both a construction and an operating viewpoint. However, neither a level bottom nor a bottom having a slope as obtained by Equation (44) is consistent with best economy of materials. Applying Mr. Thomas' Equation (63), one obtains an economical slope of 0.0286 for the 1-ft trough of Table 4 and 0.018 for the 2-ft trough. These slopes are much greater than those required by Equation (44).

Mr. Knapp points out that Equation (17) neglects the component of the velocity of the lateral inflow parallel to the axis of the flume, which component was considered in the Favre equation. In the horizontal plane no such component exists for the small channels dealt with by sanitary engineers. For such channels the inflow usually approaches at right angles to the axis of the flume. Such may not be the case, however, for dam spillways and for tail-race channels of hydro-electric power stations. Both Mr. Knapp and Mr. Rohwer suggest that an additional error is introduced by the use of the $\frac{Q}{A}$-velocity instead of the actual velocity, which varies widely over the cross-section. This appears to be the case since the actual momentum in an axial direction is somewhat greater than that computed by means of the mean velocity at the cross-section. This error is thrown into the value of f and tends to make it larger than it would be if the correction were made. Evidently, the error is small since the values of f obtained are not unreasonably large.

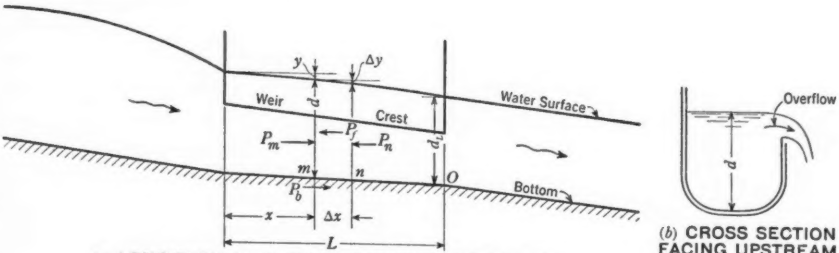
Both Mr. Knapp and Mr. Rohwer appear surprised at the apparent smoothness of the profiles of the water surface as shown in the illustrations of the paper. Although it is true that the water surface in these small channels was much smoother and had much less entrained air than is the case with dam spillway channels, the surface is always rough and transverse rollers are always present. Each point plotted on the curves to represent a measured point on the surface of the water is the average of at least two readings.

Mr. Thomas' suggested simplification of the writer's equations by the introduction of an empirical function for the depth seems quite as complicated as the writer's more rational development. The use of the empirical function, moreover, obscures the friction term in the equations. Since the friction term varies with the increment of inflow, large errors may result by the indiscriminate use of the empirical function.

It may be of interest to demonstrate how the differential equation (Equation (17)) may be applied to the analysis of the flow conditions for side weirs. The case is illustrated diagrammatically in Fig. 8. The development of Equation (17) follows in the same manner as previously presented except that for this case the lateral flow is outward and the increment (a decrement in this case) is not constant over the length of the side weir. Equation (17) becomes

$$\frac{dy}{dx} - \frac{f\,V^2}{8\,g\,R} = \frac{1}{g}\left(V\,\frac{dV}{dx} + \frac{V^2\,dQ}{Q\,dx}\right) \qquad (65)$$

in which $\frac{dQ}{dx} = q$. It is impractical because of the increased number of variables and the difficulty of integration to obtain an integral in terms of the depth for this equation. A "trial-and-error" solution may be obtained, however, in



(a) LONGITUDINAL SECTION SHOWING SIDE WEIR

(b) CROSS SECTION FACING UPSTREAM

FIG. 8

terms of the water-surface profile, in a manner similar to that suggested by Professor Howland for the lateral spillway channel.

Equation (65) may be written

$$dy = \frac{f}{4\,R}\,\frac{V^2}{2\,g}\,dx + \frac{V}{g}\,dV + \frac{1}{g}\,\frac{Q}{A^2}\,dQ \qquad (66)$$

This equation may be integrated approximately over the short distance Δx as follows:

$$\Delta y = \frac{f}{4\,R_{av}}\left(\frac{V^2}{2\,g}\right)_{av}\Delta x + \frac{V_n^2 - V_m^2}{2\,g} + \frac{Q_n^2 - Q_m^2}{2\,g\,(A^2)_{av}} \qquad (67a)$$

or

$$\Delta y = \frac{f}{4\,R_{av}}\left(\frac{V^2}{2\,g}\right)_{av}\Delta x + \frac{V_n^2 - V_m^2}{2\,g} + \frac{\Delta Q(Q_n + Q_m)}{2\,g\,(A^2)_{av}} \qquad (67b)$$

The design may be started at the down-stream end (point o) where d_L, A, and Q are fixed to suit the requirements of the problem. For an assumed value of Δx and a corresponding value of ΔQ to suit the conditions of the problem, a trial position of the weir crest may be set for the down-stream end, and an estimate made of the value of the first Δy. After suitable adjustments are made, the process may be continued for succeeding values of Δx up stream until a length of weir is obtained which will deliver the required overflow.

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DISCUSSIONS

LARGE CORE DRILLS AID CONSTRUCTION AT CHICKAMAUGA DAM

Discussion

BY MESSRS. J. G. TRIPP, JAMES B. HAYS, AND O. N. FLOYD

J. G. TRIPP,[4] M. AM. SOC. C. E. (by letter).[4a]—The difficulties encountered in sinking relatively small shafts in the over-burden, broken, and solid sedimentary rocks at Chickamauga Dam are outlined clearly in this paper. The methods chosen by the engineers, and the costs achieved, deserve commendation.

There were probably three methods open to the builders to complete satisfactory foundations under the lock guide wall:

(1) The shot-drill and casing method (that chosen);

(2) The standard "air" method; and

(3) The rotation water method, more or less successfully used in very deep foundations in several difficult locations in and around New York, N. Y.

It appears clear that method (1) was chosen on account of a desire to explore the ground, equipment availability, and operating personnel. If the costs given are the actual totals, there is little doubt that the choice was wise. Under standard "air" conditions, costs would have approached twice those achieved. The rotation water method, if available, would have cost about $20 per ft.

However, the third method deserves mention. Shafts averaging 55 ft with a maximum depth of 82 ft have been driven successfully in Manhattan, where broken rock, boulders, quicksand, and rip-rap were the materials encountered. Also caissons 8.5 ft by 110 ft were sunk successfully across the Connecticut River.

This method consists of a caisson rotating in a column of water. The water enters the caisson under controlled pressure and the lighter and smaller materials pass under the bottom of the cutting edge of the steel shell and appear at the surface annularly about the caisson. Those elements too large

NOTE.—This paper by James S. Lewis, Jr., Assoc. M. Am. Soc. C. E., was published in June, 1939, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: October, 1939, by J. B. Newsom, Esq.

[4] Pres., Tripp Constr. Corporation, New York, N. Y.

[4a] Received by the Secretary October 17, 1939.

to pass are left in the caisson and removed from time to time, as required. A properly designed cutting edge cuts into the solid rock for any desired distance, say, to 4 ft, when rotation is stopped; the materials surrounding the caisson are then allowed to settle and consolidate, and the caisson becomes ready for unwatering and for concrete. This method obviates the necessity of grouting except in the bottom rock should conditions require. Standard methods accomplish such grouting and a safe steel-cased shaft is ready for steel reinforcement and concrete.

In conclusion, if relatively solid rock had existed from the grass roots down, no shafts would have been needed. In the case at Chickamauga Dam the engineers were faced with the common ailments of all limestone dam sites; that is, corrosion, eroded contacts, and redeposited, tilted, folded, and faulted strata with consequent water problems. In such ground little useful knowledge can be obtained from ordinary core borings. In such cases, the large core drill is indispensable, because an inspection well that the geologist or foundation designer can actually go down into gives all necessary predesign information with certainty. Finally, it would appear that the difficulty of ascertaining foundation conditions under the guide wall at Chickamauga lock led inexorably to the choice of method described.

JAMES B. HAYS,[5] M. AM. SOC. C. E. (by letter).[5a]—One of the most difficult operations necessary to secure a satisfactory foundation at the Chickamauga Dam is described in this paper. The author demonstrates the fact that exploration in limestone rock must be thorough.

The original borings were not spaced closely, and in the area covered by the work described by the author there was only one hole directly under the wall, with two or three close by. It was realized that the foundation was not particularly good; but it was not until work had been started and excavation for the lock walls was well under way that the real situation developed. Preliminary borings were put down at the corner of each pier, and the detailed information, with other experience on the project, indicated that open excavation to rock would be practically impossible. The coffer-dam was too close to allow open cuts without timbering. Timbered shafts would have exposed too much surface to the mud and clay seams which, in time, would have required excessive bracing and involved considerable leakage. As a result the design was revised, as shown, to support the piers on reinforced concrete columns on rock, since most of the equipment necessary was already available.

Mr. Lewis was placed in charge of this particular feature which was one continuous battle from start to finish against water and mud, and the danger of blows. However, when finally ready for concrete, the holes were in excellent shape so that the leakage caused little, if any, trouble in concreting.

Evidently the cooling of concrete placed in the columns was successful. The writer recalls that one column was shortened more than $\frac{1}{8}$ in. by the cir-

[5] Constr. Engr., Kentucky Dam, TVA, Gilbertsville, Ky.

[5a] Received by the Secretary November 6, 1939.

culating cold water. This work was done during the late winter months; hence the river water was used for cooling.

One lesson that was demonstrated on this project was the fact that exploration holes in limestone formation should be spaced closely and drilled accurately; and they should be carefully "logged" by a competent geologist. A second lesson was that grouting of coffer-dams in such a formation was necessary. The work described by the author was in the first of three stages of river coffer-dams. Coffer-dam grouting was started too late in the first stage to be a complete success, although it did benefit to some extent. The second and third stages were grouted thoroughly and, as a result, were remarkably dry. During the construction period, the geologist, Portland P. Fox, was of great assistance in defining the detailed rock structure. Fig. 3 is one of many that were used.

The large core drills were useful for checking the results of grouting and foundation treatment for shafts and pump sumps.

O. N. Floyd,[6] M. Am. Soc. C. E. (by letter).[6a]—A rather complete account of a difficult piece of work, planned and executed with very limited guidance from precedent, is presented in this paper. The frankness with which the limitations of the equipment and the failure of certain attempted methods are treated adds materially to its value.

The writer wishes to call particular attention to the added value of these large holes to the engineer, in that they make it possible to see the rock in place. The bedding planes, thickness of cored strata, and any bentonite or clay seams or open cavities are all there to be examined and probed. It was quite helpful, in the work described by the author, to be able to see the rock on which the columns were to rest and also to judge better the lateral support the columns would have because of the developed thickness and probable extent of the large blocks of limestone penetrated on the way down.

The 36-in. holes were helpful in determining the extent and nature of some bentonite seams under parts of the main walls of the lock at this same job. Furthermore, in the cut-off under the earth wings of this dam, where much bad rock had to be removed and a few deep caverns had to be cleaned out and filled with concrete, the 36-in. holes were used for working shafts during the cleaning operations and later for refilling with concrete. Both the cost and the time for sinking shafts in this manner were very favorable.

Another place illustrating the superior value of the larger holes was at one of the early small core drill holes under the south earth embankment of the Chickamauga Dam which indicated a rather deep nest of boulders, clay, and gravel. A 36-in. hole sunk to include this hole showed that the small drill had gone down about a foot away from the almost vertical face of a solution channel in the rock and, since several of the more resistant strata were protruding from this face, they had been cored or broken off and classed as boulders in the record of the first hole.

[6] Cons. Engr., Dallas, Tex.

[6a] Received by the Secretary November 13, 1939.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

WIND BRACING IN STEEL BUILDINGS

SIXTH PROGRESS REPORT OF SUB-COMMITTEE NO. 31 COMMITTEE ON STEEL OF THE STRUCTURAL DIVISION

Discussion

BY ETHAN F. BALL, M. AM. SOC. C. E.

ETHAN F. BALL,[43] M. AM. SOC. C. E. (by letter).[43a]—Structural designers will find this Report an interesting and valuable addition to their library of wind bracing literature, particularly Division (*A*).

For building frames of the usual type, nearly all designers, including the writer, have applied the cantilever and portal methods, presented by Robins Fleming [31] in 1913, to determine wind stresses. These methods give results which may be far from correct, due to the assumption that the various members of the bent will take certain arbitrary proportions of the wind stress without regard to their size or stiffness.

Professor Witmer's method, presented in Division (*A*), is to determine wind reactions based on actual stiffness of the members. The Report explains how the reactions may be obtained quickly without resorting to the use of the slope deflection, or other "exact" methods which require too much labor to be practicable. The reaction-ratio curves given in Fig. 2 are simple to use, but their application is limited to symmetrical four-column bents. Most building bents are either unsymmetrical or contain more or less than four columns. Professor Witmer has developed the ingenious method of *K*-percentages for determining reaction ratios for such bents, and this method, therefore, can be used for all buildings of the usual height and type.

After finding the wind-reaction ratios, all wind stresses may be determined easily by a statically determinate process. Division (*A*) of the Report shows

NOTE.—The Sixth Progress Report of Sub-Committee No. 31, Committee on Steel of the Structural Division, was presented at the meeting of the Structural Division, New York, N. Y., January 19, 1939, and was published in June, 1939, *Proceedings*. Discussion on this report has appeared in *Proceedings*, as follows: September, 1939, by Messrs. Arthur G. Hayden, Robins Fleming, and C. M. Goodrich; and November, 1939, by Messrs. Samuel T. Carpenter, and Rolland A. Philleo.

[43] Asst. Chf. Engr., Bethlehem Steel Co., Fabricated Steel Constr., Bethlehem, Pa.

[43a] Received by the Secretary November 10, 1939.

[31] "Wind Bracing Without Diagonals for Steel-Frame Buildings," by Robins Fleming, *Engineering News*, Vol. 64, March 13, 1913, p. 492.

that the results obtained by the Witmer method conform quite closely with the results obtained by "exact" methods. The accuracy obtained seems to be as close as is warranted, considering the usual assumptions: (1) That the steel frame alone carries all the wind load on a building, regardless of the stiffness and strength of partitions, floors, and walls; and (2) that allowable unit stresses are increased by about 25% if wind load raises the total stress in a member by at least that percentage.

In irregular bents, such as offset bents or bents with interior columns and girders omitted, the members at these points will vary from the average members, and it is possible that the results of the Witmer method will vary from those of the "exact" methods accordingly. This is indicated by the comparison of results given for the five-bay, twenty-six-story, symmetrical bent, regarding which the Committee concluded: "It is believed that the rather large differences for some members may be attributed to the irregularity of the frame" (see heading, "Five-Bay, Twenty-Six-Story, Symmetrical Bent").

In set-back bents, all the approximate methods encounter some trouble at the point of set-back, because it is not practicable to use girders stiff enough to put the proper stress and elongation in the columns which are cut off. One solution of the problem is to divide the offset bent into two rectangular frames, one above the set-back and the other below: Design the upper frame for the wind loads above the set-back; and design the lower frame for the loads below the set-back, plus all the wind load above the set-back applied as a horizontal load at the top of the lower frame. To the proper columns in the lower frame must be added the reactions of the columns in the upper frame. These reactions will produce direct deformation in the columns, which will require secondary corrections to the primary girder shears as first computed for the lower frame.

The writer believes that Professor Witmer's method will come into general use as soon as structural designers become familiar with its advantages. It is more accurate than the old approximate methods, and it produces results just about as quickly. In using the method for tall, slender buildings, engineers should not overlook direct deformation in columns, the effect of which, on wind stresses, is explained in Division (*B*) of the Report.

The torsional effects of wind have not usually been considered in the design of building frames, and it is probably not necessary to consider them for most frames. However, the necessity for considering torsion, in the case of buildings of odd shape, is definitely indicated by the results of the study made by the Committee on a building of extreme proportions.

In view of the uncertainty regarding the amount of wind force on high buildings, the writer would prefer to start increasing the uniform load of 20 lb per sq ft at the 300-ft level instead of at the 500-ft level as recommended by the Committee.

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DISCUSSIONS

FIELD TESTS OF A SHALE FOUNDATION

Discussion

BY HARRY H. HATCH, M. AM. SOC. C. E.

HARRY H. HATCH,[4] M. AM. SOC. C. E. (by letter).[4a]—Considering the individuality of every foundation material or its constituent soil particles, the difference in location and climatic conditions, the manner of, and personal equation in, investigation, and the variation in the experience, competence, and viewpoints of the investigators, one cannot be surprised that "consistency of test results," reported in this paper, is lacking.

The apparently negative result of a conscientious test is just as important as if its results agreed one hundred per cent with the preconceived notions. It is desirable to standardize tests and testing methods for the sake of economy of time and money; but on no job should an engineer be content with the "patent medicine" laboratory theories and notions. He should explore further, by independent research, to find the truth for the case in hand.

Only recently have the rule-of-thumb methods been discarded and scientific investigation applied to these jobs. Sufficient data are not available yet even to suggest a law of averages. It is essential that all such investigations should yield concise information in regard to the purpose and the method of conducting any test that is not well known or standardized. It is all the more important, therefore, to have before the profession the accurate description and results of every investigation of this nature in order to augment the store of information and knowledge on the subject; and the author should be commended for his purpose and efforts in presenting this paper.

Under the heading "Laboratory Tests of Shale," Items (*h*) and (*i*) do not entirely fail to conform with Items (*k*) and (*l*).

$$s = \frac{W}{A} \tan \phi + C \qquad (4)$$

NOTE.—This paper by August E. Niederhoff, Assoc. M. Am. Soc. C. E., was published in September, 1939, *Proceedings*. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion of the paper.

[4] Div. Engr., Springfield Water Works, Springfield, Mass.

[4a] Received by the Secretary October 25, 1939.

in which: s = unit shear; $\frac{W}{A}$ = normal unit load = external load per foot of area; ϕ = angle of internal friction; and C = unit cohesion.

The computed shear value for Item (h), according to Equation (4), is 7.2 tan 45° + 4.5 = 11.7 tons per sq ft, or 88.9% of the observed value, 13.2; and similarly for Item (i), it is 3.6 tan 45° + 4.5 = 8.1 tons per sq ft, or 97% of the actual value, 8.35. Items (k) and (l), therefore, give values about 11% and 3% less than the actual; and unless this consistency is purely by coincidence, the test results cannot be considered seriously in error.

The writer's experience has been that the coefficient of friction (and hence the angle of internal friction) is not constant for a given soil, and that it decreases with the increase of pressure. If C, the cohesion, is kept constant at 4.5 in order to have the actual and the computed shear values agree completely, the angle of internal friction should be 50° 23′ and 46° 55′ for Items (h) and (i), respectively, in this case. That is, the higher the pressure the larger the angle of internal friction will be, which does not agree with the writer's contention. Could the solid rock formation and its bedding planes account for this? On the contrary, with due respect to the difference in their normal loads, the actual shearing value parallel to bedding plane is higher than that normal to bedding plane, instead of vice versa as would be expected. Perhaps, after all, bedding planes in this solid rock may not be of any consequence during the shearing tests; or the value given to cohesion may be responsible for the inconsistency. More details as to the number of tests conducted for the two planes, with their individual results under different pressures and with a description of the method of averaging them, will be necessary to interpret the given averages intelligently.

There are only two average shear values given. More would have been desirable, but these two will satisfy Equation (4) with the assumed constants ϕ = 53° 25′ and C = 3.5 (see Fig. 10(a)). A description of the author's methods of testing cohesion and the angle of internal friction of a solid rock formation will be of interest and may throw some light on the subject.

Equations (4) and (3) are identical. In the latter $x = s$, the shearing force in tons per square foot; and $y = \frac{W}{A}$, the normal pressure in tons per square foot. Therefore, $C = 3.7$, and tan $\phi = 0.9$, or $\phi = 42°$. As already stated, ϕ cannot remain constant as the pressure increases, even if shearing value increases with the increase of pressure. However, this relation does not follow the locus of a straight line, as given by Equations (3) and (4). Either equation is only approximate and, when obtained from the plottings of only two or three points, the equation becomes entirely inapplicable. In Fig. 10(b), Curve A gives the relation between Columns (2) and (5) of Table 1; Curve B is the plot of Item No. 2, Table 2; and C is the plot of Equation (3). According to Equation (3), or Curve C in Fig. 10(b), there are 3.7 tons per sq ft of cohesion between the concrete and the shale. Can there be any such cohesion between the concrete and the shale blocks? With no cohesion existing between the blocks, the extension of Curves A and B down to the origin can well be justified.

From Fig. 10(*b*), it is perfectly evident that the relation between shear and normal pressure can be represented neither by a straight line, nor by a simple exponential equation, over the entire range.

From a practical point of view, what necessity is there for cohesion tests and angle of internal friction tests when actual shearing tests are likely to give a much better picture of the existing conditions in the structure?

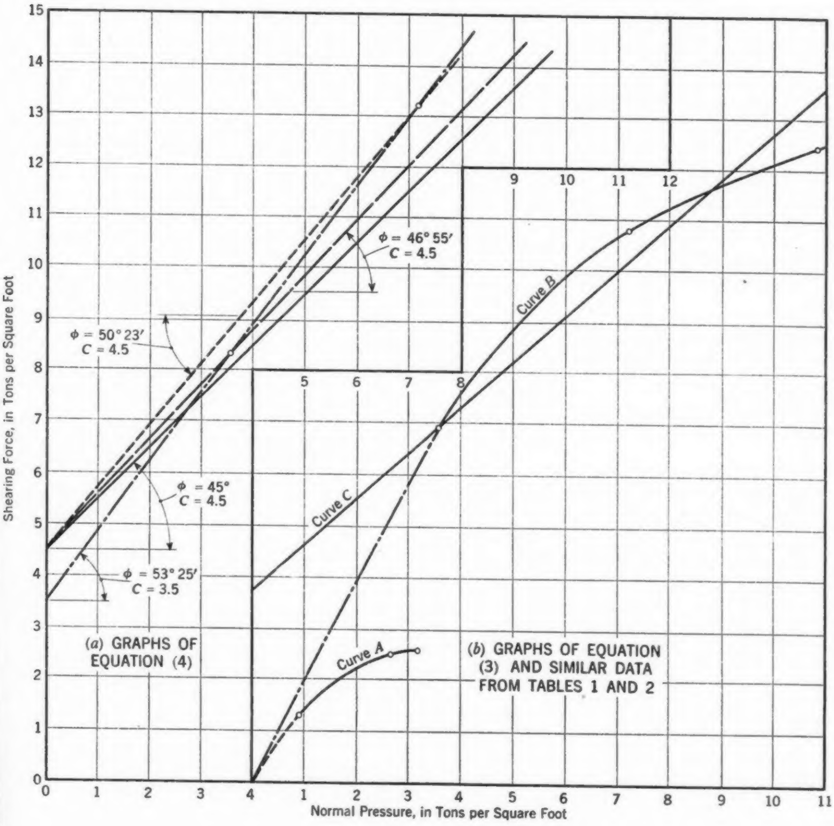

Fig. 10

It is preferable to have as many direct shear tests as possible under the circumstances, but a sufficient number of them at any rate, and to plot them all in order to obtain a more or less representative graph showing the relation between the shear and normal load, without paying any attention to the uncertain term of the angle of internal friction of a solid rock formation or its cohesion.

As stated by the author, the asphalt coating separating the concrete and the shale must have acted as a lubricating plane between the two and was mostly responsible for the lower shear and coefficient of friction values. Due

to the asphalt alone, disregarding any other possible reason, it is questionable whether the two sets of test results can be considered comparable.

Although the two sets of tests may be unlike in more ways than one, the behavior of the coefficient of friction in both cases is according to the writer's experience; namely, the coefficient of friction is not constant, but decreases with the increase of vertical pressure. Insufficient data are available, however, to produce for this job a representative graph or equation expressing the relation between the coefficient of friction and the vertical load. Any such relation obtained from two or three tests would be practically of no value.

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DISCUSSIONS

THEORY OF LIMIT DESIGN

Discussion

BY L. H. DONNELL, ESQ.

L. H. DONNELL,[58] ESQ. (by letter).[58a]—This paper is an important contribution not only to the field of structural analysis, but to fundamental engineering philosophy.

Every one knows, but sometimes needs to be reminded, that the real object of engineering design is not to insure that stresses remain below a certain limit, but to build the most efficient structures and machines that will perform, satisfactorily, the functions for which they are intended. The method of computing stress distributions under purely elastic conditions, and comparing these with the stresses that will cause some elastic buckling, or other type of "failure," has been, and remains, a most useful tool for this purpose, but designers have long been familiar with cases in which its strict application would be not only very difficult but also highly misleading. This paper demonstrates that in other cases where designers have considered the elastic approach the most advanced method possible, another method may be simpler and may give a more accurate measure of the capacity of structures to perform their real functions satisfactorily.

Undoubtedly, this entire subject has been studied too little, with the result that there are few who have a clear, well-thought-out understanding of it. Designers should not be afraid of such questioning and reviewing of engineering fundamentals, and surely there can be no better place for this than the national forums of the great engineering societies.

The method in question is one which all designers have long accepted in certain fields of engineering, and which in essence has been accepted in some other countries in the structural field. Professor Van den Broek has made important contributions to its development, particularly in generalizing it to in-

NOTE.—This paper by J. A. Van den Broek, M. Am. Soc. C. E., was published in February, 1939, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: May, 1939, by Messrs. John H. Meursinge, I. K. Silverman, Edward Godfrey, Basil Sourochnikoff, E. Mirabelli, C. M. Goodrich, George Winter, and Francis E. Simpson; June, 1939, by Messrs. Joseph A. Wise, Alfred M. Freudenthal, Hans H. Bleich, Alfred S. Niles, and A. Floris; September, 1939, by Messrs. L. H. Nishkian, and F. G. Eric Peterson; October, 1939, by E. S. Fabian, Assoc. M. Am. Soc. C. E.; and December, 1939, by A. A. Eremin, Assoc. M. Am. Soc. C. E.

[58] Associate Prof. of Mechanical Eng., Armour Inst. of Technology (formerly in chg. of Stress Analysis and Structural Research, Goodyear-Zeppelin Corporation), Chicago, Ill.

[58a] Received by the Secretary November 24, 1939.

clude compression members of trusses and other parts subject to buckling. His most important contribution, however, is in bringing to the attention of the profession, and helping to clarify, some of the gaps and inconsistencies in engineering reasoning.

It is quite possible, of course, that he goes too far in some ways in his exposition of the method, as is natural and perhaps necessary when one is combating ingrained habits. Thus it may be, as he states, that it is excessive deformation which really marks the limit of usefulness of structures. What the designer is really interested in, however, is practical means for predicting this limit of usefulness; and it is to be noted that the author does not propose to calculate this limit (and would indeed find it very inconvenient to so calculate it) by a study of the deformations of the structure. Rather he calculates it by considering the way in which the forces distribute themselves through the structure, just as in any elastic theory.

It would seem that the author does demonstrate, beyond a doubt, the following: He shows that the forces that first produce yielding or buckling in certain parts of a structure, while they may indicate the limit of strength of these parts, do not necessarily indicate the limit of strength of the structure as a whole, when these parts are redundant. He shows that when these parts are capable of yielding sufficiently, while still maintaining more or less undiminished resistance, the limit of strength of the structure itself is not reached until all the redundant parts have reached their individual limits. He shows also that this limit of strength of the structure is easily found and in many important cases differs an important amount from the strength usually calculated by much more laborious methods. He shows, furthermore, that the deformations of the parts and of the structure as a whole remain "of the order of magnitude of elastic deformations" until the limit of strength of the structure is reached.

Two things remain which do not seem to be clearly demonstrated—namely, that such deformations of the structure as a whole may not be great enough to destroy its usefulness, and that such deformations can be borne by the parts involved while still maintaining more or less constant resistance. It is no criticism of the paper to say that these questions are not adequately covered, because obviously one paper cannot be expected to cover all sides of such a subject; but it seems evident that these questions must be studied thoroughly before the method can be acceptable for application in cases where proof loading is impossible.

For most types of structures the possibility of having permanent deformations of this order of magnitude under "once-in-a-life-time" load conditions is probably not serious, and a decision as to whether this is permissible can be given offhand. In this connection it should be noted that the large deformations of steel buildings, bridges, etc., which have sometimes been produced by hurricanes or floods, are undoubtedly produced after all redundant members have reached their limits and not before.

The question of whether individual members of structures can "give" the amounts required without weakening is more difficult and requires quantitative

studies of (1) the amount of giving which would be required in different types of structures, and (2) the amount which various types of members can give without weakening. Considering the first question, it is obvious that the amount of giving required will depend very much upon the type of structure, and in many cases may be very much more than maximum elastic deformations. For example, in Fig. 43, the amount bar a would have to stretch plastically before bars b would reach their yield point would depend on the angle of bars b, and would be $\left(\frac{b}{a}\right)^2 - 1$ times the maximum elastic strain, if the bars are all of the same material; or, in Fig. 44, the plastic strain in the tie bar a must be large before the stresses in the truss reach the yield point if the dimensions a and b are small compared to c.

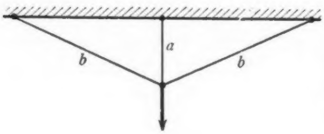

FIG. 43

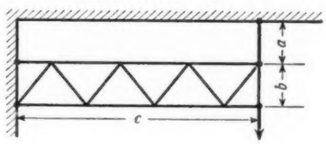

FIG. 44

Considering question (2), studies must be made of the relation between the longitudinal resistance of struts and the longitudinal deformation during local or Euler buckling, and of the relation between the bending resistance of deep beams and the bending deformation when local or lateral buckling of the compression flange occurs. There have been many theoretical and experimental studies of buckling problems in both the elastic and plastic range; but, heretofore, this particular phase of the question has not been considered important and has apparently escaped investigation.

As to members under tension, although it is true that plastic strains can be a hundred times or more as large as elastic strains in a mild steel tensile test specimen, it does not necessarily follow that this is true in structural tension members as they are commonly made, which frequently includes reductions of area and stress concentrations at the ends. To be sure, there is a powerful antidote against the obvious tendency in such cases for most of the strain to concentrate in these short, weakened sections—namely, the phenomenon of strain hardening, which stiffens up such weak points after they have yielded somewhat. However, obviously there are limits to the extent to which this favorable tendency can overcome the unfavorable, as is observed in tensile test specimens after the necking-down stage is reached. Hence some quantitative studies of this question are needed. This is especially important in view of the present tendency toward the use, for greater efficiency, of some of the higher strength materials, which almost always have lower ductility.

It should be emphasized that such studies are important not merely in order to make some new method of analysis possible, but chiefly because of the possibility which they present for making improvements in the actual ultimate strength of present types of structures through minor modification of details. Thus it might be found that certain types of tension members, although strong

enough elastically, will not stand the plastic strain required to realize the full ultimate strength of the structure, and that this ultimate strength can be obtained by changing to tie-rods with enlarged ends, such as are frequently used now for economy of material and resistance to shocks. Such possibilities for increasing efficiency would not be suggested by present elastic methods of analysis.

If such studies show that no modifications are necessary in order to realize the increased strength indicated by the new method of analysis—that is, if present-type structures are really stronger than designers have been computing them to be—then the factors of safety to be used in applying the new method require discussion.

The factors of safety used in many branches of engineering are based, at least in part, on practical experience of the past in similar work. If they were too low in the beginning, too many failures occur, forcing engineers to raise them; or, if they were higher than necessary, in time some daring pioneers are certain to try lowering them, and if their experiments are successful other engineers eventually copy them. Thus values are gradually evolved which, at least on the average, are reasonably close to what they should be.

Obviously, this process is valid only when the same methods of analysis are used on the new and old designs. If engineers do not improve the type of design, and introduce a new method which gives more optimistic estimates of the strengths of structures, factors of safety based on past experience should presumably be raised enough so that structures designed by the new method will be as strong, on the average, as those designed by the old methods.

This does not at all mean that the new method would give the same results in individual cases as the old, and hence that it has no advantage other than its greater simplicity. On the contrary, a design method which gives an imperfect measure of the real useful strength of a structure, as elastic methods evidently may, can obviously result in too low a real strength in some cases and too high a real strength in others, even if it is regulated to give correct results on the average. Conversely, any method which gives a better measure of the real useful strength, as the one under discussion evidently may, will always give results closer to what they should be in individual cases; and, of course, in so far as this represents decreased uncertainty in design, the factor of safety can be reduced somewhat, counterbalancing, at least partly, the necessity for raising it.

One other point is suggested by this paper: If there is any real objection to relying on the yielding of some members of a structure under very extreme conditions, the same ultimate strengths could be obtained, without the necessity for such yielding, by the expedient of introducing initial stresses into the structure when it is built, such that under the worst conditions all redundant members would reach their ultimate loads at the same instant. Practical means for introducing such initial stresses could probably be devised if it proved worth while. Such an expedient, of course, could be fully applied only in cases where it can be assumed that loadings are predominantly in one direction.

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DISCUSSIONS

RECONSTRUCTION OF THE WALPOLE-BELLOWS FALLS ARCH BRIDGE

Discussion

By Messrs. Conde B. McCullough, and H. M. Nelson

Conde B. McCullough,[6] M. Am. Soc. C. E. (by letter).[6a]—Congratulations are due the authors, not only on a most coherent presentation, but also on an exacting piece of reconstruction, efficiently and safely performed. As aptly emphasized in their report, jacking operations on major structures are normally conducted under conditions of rigid stress control, whereas in this case the distribution of stress at the beginning of jacking operations was highly problematical. The care which they exercised in controlling both the magnitude and the position of the applied thrust leaves little to be desired.

It is gratifying indeed to see a "repair job" of this character accorded the distinction of a detailed technical write-up. In general, repair work has earned few plaudits, notwithstanding the fact that its technique often calls for design and erection procedures more complicated and involved than in new construction. Moreover, marked economies are generally effected; and in the present instance, the expenditure of approximately $120,000 operated to save the cost of a new structure which the authors point out would have amounted to several times this sum. Economies such as this are all too frequently lost sight of in the enthusiasm for new construction.

An instance which occurred many years ago involving the Morrison Street Bridge in the City of Portland, Ore., illustrates the economies which are sometimes possible. This structure consisted of a 384-ft draw-span flanked on one end by a 205-ft fixed truss span and on the other by two spans at 267 ft. In 1919 this structure had apparently been outgrown by its traffic, and was in a rather unfortunate condition of disrepair. The floor system was inadequate, the eye-bar chains over the draw-span towers were seriously overstressed, as were most of the compression members in the bottom chord with the span open, and popular opinion had it that the bridge was outmoded, inadequate,

Note.—This paper by Messrs. H. E. Langley and Edward J. Ducey was published in April, 1939, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: September, 1939, by J. B. French, M. Am. Soc. C. E.; and October, 1939, by Lewis D. Rights, M. Am. Soc. C. E.

[6] Asst. State Highway Engr., State Highway Dept., Salem, Ore.

[6a] Received by the Secretary December 5, 1939.

and unsafe. After a rather complete investigation, the writer recommended a system of repairs estimated to cost $208,000, based upon the following economic comparisons:

(*a*)	Estimated cost of repairs.................	$ 208,000
(*b*)	Estimated total maintenance expense for ensuing 12-yr period	111,000
(*c*)	Total cost during ensuing 12-yr period for repairs, renewals, replacements, and ordinary maintenance (Item (*a*) + Item (*b*)).	$ 319,000
(*d*)	Annual interest charge on capital to reconstruct ($2,500,000 at 5%).............	125,000
(*e*)	Total interest expense for 12-yr period.....	1,500,000
(*f*)	Financial saving realized by repairing old structure (Item (*e*) − Item (*c*))	$1,181,000

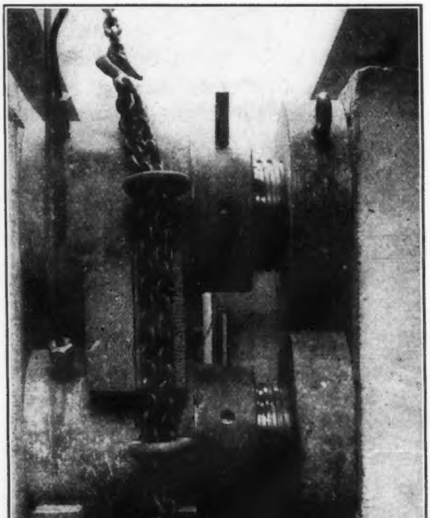

Fig. 15

This report was finally adopted, the repairs made at a cost of slightly more than $200,000 and the structure is still carrying unlimited and unrestricted traffic after an extended service period of nearly twenty years. This instance is typical of many wherein distinct economies accrue from the maintenance of old construction even if the repair work involved is complicated and somewhat hazardous.

In connection with the adopted jacking procedure, the authors mention the extreme sensitiveness of the system, the slightest variation in pressure on the jacks being instantly discernible in the device for measuring cable stretch. They also mention a rather peculiar difference between the action of the tower jacks at the two ends, involving an automatic drifting of the far jacks with little or no pumping. These two qualities—extreme sensitiveness and an inclination toward erratic behavior—seem to be somewhat characteristic of all hydraulic jacking setups.

In 1931 and 1932, the writer had the opportunity of experimenting with an hydraulic jacking system in connection with the decentering of the highway bridge over the Rogue River in Oregon. This structure consists of a group of

seven 230-ft reinforced concrete arch spans, and the decentering was performed in accordance with the Freyssinet method, by means of an hydraulic system, consisting of a battery of four 275-ton jacks installed at the crown of each rib. During decentering operations, observations were made of: (1) Total jack thrusts and moments; (2) crown openings at extrados and intrados; (3) vertical

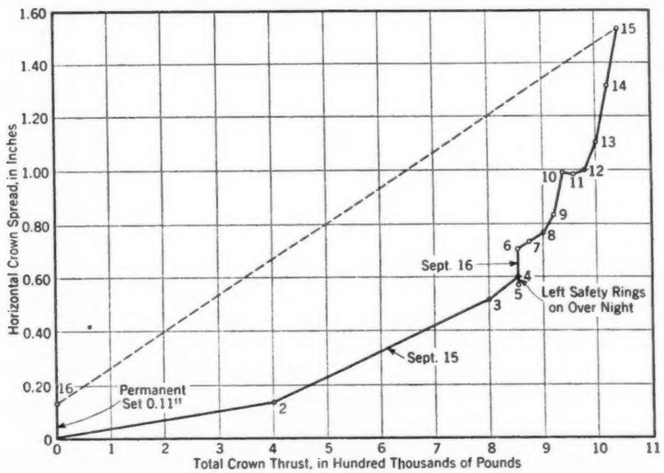

FIG. 16.—SPAN NO. 1; FIRST RUN, SEPTEMBER 15 AND 16 (CENTERS STILL IN PLACE)

movements at crown, spring line, and intermediate points; (4) internal strains at extrados and intrados at five sections in each span; (5) rotation of rib axis at crown, quarter points, skewbacks, and piers; (6) horizontal movements at spring line; and (7) temperature and shrinkage.

Jack pressures were measured by calibrated gages, crown openings with a sliding scale and vernier, vertical movements by means of a system of water levels, horizontal movements by weighted invar wires, and internal strains by an installation of electric telemeters. The hydraulic jacks which were manufactured especially for this job by the manufacturer, who had previously designed and constructed similar equipment for M. Freyssinet, are shown in Fig. 15.

A graph (Fig. 16) shows the relation between crown thrusts as measured by the jacks, and the horizontal crown spread. It will be noted that as the jack thrust was increased the rate of crown spread increased gradually to Position 4. This increase in rate was due to the fact that, as more and more of the load was taken by the arch, the reaction of the centering against the ribs became correpondingly less, with the result that the frictional resistance to rib shortening was decreased. In Position 4, the safety rings were tightened, and the arch was allowed to stand overnight on the jacks, during which period the crown gap closed 0.03 in. due to a tightening up of the safety rings. When the decentering thrust was again applied, an additional crown spread of 0.1 in. was observed without increase in thrust. A shortening of 0.02 in. can be accounted for by the intervening drop in temperature; and the residue represented restrained

shrinkage in the concrete taken up as the arch was lifted clear of its centers. This graph, typical of a large number of similar ones for the various spans, indicates the marked sensitiveness of the system, very slight temperature and shrinkage effects being readily detectable on the pressure gages.

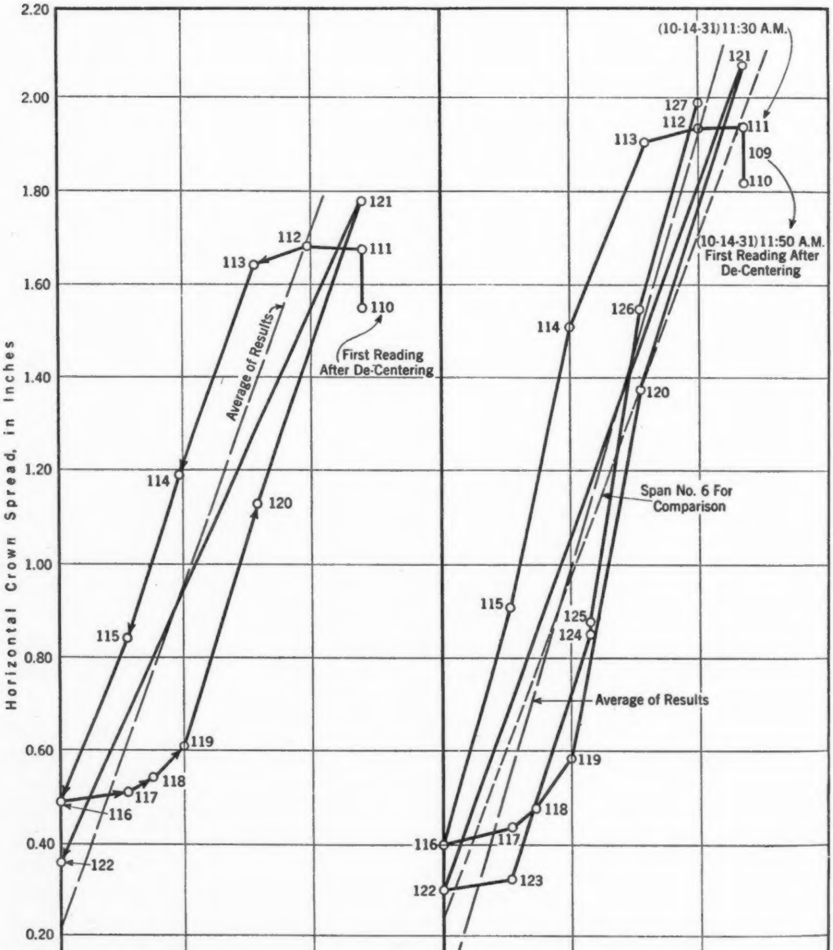

FIG. 17.—SECOND RUN, OCTOBER 14 (CENTERS REMOVED)

Another interesting phenomenon is indicated in Fig. 17 which contains graphs of crown thrust *versus* crown spread for Spans 6 and 7. It will be noted that whenever the direction of motion was reversed, a change in thrust of about

70 000 lb was necessary to reverse the motion. This rather puzzling condition persisted throughout the entire range of these experiments, and was closely checked by the telemeter readings as well. It was thought at first that jack friction might account for it, at least partly; however, a careful calibration of the jacks indicated no frictional resistance within the limits of accuracy of the gages. The only other logical explanation offered was that the joints and articulations in the spandrel structure, although theoretically free to move, possessed a distinct and fairly constant resistance to reversing motion.

The writer was reminded of this rather perplexing action when he noted the authors' description of the drifting of the distant tower jacks during the adjustment of the Walpole-Bellows Falls structure. Contradictory and confusing phenomena are not infrequent in hydraulic adjustment setups of this character.

All in all, this paper constitutes a timely and valuable contribution to the literature. More attention should be paid to the scientific design and execution of repair work. Furthermore, it would appear that there is a greater field of utility for hydraulic jacking adjustments than is evidenced by present usage. The "weighing in" of reactions on draw-spans and on continuous trusses and girders not only has the advantage of greater accuracy but also operates to eliminate any stresses that may be thrown into the superstructure through elastic support displacements or a plastic "take-up" in the foundations. For extensive installations of this kind it would appear that there are four independent methods for stress determination which may be used as a mutual check. These are: (1) Strain measurements by means of a system of extensometers, telemeters, or other suitable device; (2) deflection measurements, including angular rotation measurements at joints and at critical sections of the structure under adjustment; (3) measurement of reaction displacements at the jacks; and (4) gage readings on the jack-pressure lines.

The paper will undoubtedly serve a most useful purpose in stimulating interest in this somewhat complicated phase of construction endeavor.

H. M. Nelson,[7] Assoc. M. Am. Soc. C. E. (by letter).[7a]—An ice jam in the Connecticut River, composed of very large, thick cakes, which had been stalled for about four days at a point 6 miles above the Walpole-Bellows Falls steel arch bridge, passed through the bridge in the afternoon and evening of March 17, 1936. As the surface of the water and ice at that time was only slightly above the concrete protection which was placed in front of the skewbacks and extended beyond the first panel points, when the dam just below the bridge was raised some years ago, the bridge steel suffered no damage during the passage of the heavy ice jam. After the passage of the ice jam the river continued to rise, and, at about 1:00 a.m. on the morning of March 19, a crash at the bridge was heard. It is probable that the damage to the New Hampshire end of the bridge occurred at that time and that it was caused by a large ice floe. Because it was dark there were no eyewitnesses. The water was then at about Elevation 121.6.

[7] Hydr. Engr., New England Power Service Co., Boston, Mass.

[7a] Received by the Secretary December 6, 1939.

During the early forenoon of March 19, when the Connecticut River was at Elevation 124.6, and near the peak of its flood, a floe, composed of ice that had been thrown out on a meadow during the run of the ice jam (or that had floated out of some bay that had not broken up when the ice jam came through), was seen approaching the bridge rapidly. The river bends sharply to the left in the vicinity of the bridge, and the floe was carried to the Vermont side in such a manner that the blunt front end of it struck the lower chord of the up-stream arch near the second panel point. This floe appeared to have an area of a quarter of an acre or more. Just previous to the breakup, the river ice had been found to be 18 to 24 in. thick. Assuming it to be an average of 21 in. thick, the floe must have had a weight in excess of 500 tons. The velocity of the river above the bridge was about 4 miles per hr. The impact caused the entire structure to vibrate so violently that the writer, who was leaning over the bridge sidewalk railing to observe, was thrown to the floor. When the ice floe swung around toward the center of the river and passed along, it could be seen that the lower chord had a sharp offset about equal to its width. The actual damage to the New Hampshire end of the bridge was not discovered until the next day when the flood waters had receded somewhat.

The decision to reject the first plan of repair which contemplated placing a large number of pile bent supports in the river channel was a wise one, as was proved later when, in January, the ice in the river broke up over a distance of nearly 12 miles and came to rest only about 2 miles above the bridge. At this time the bridge was supported on the temporary pile bents driven at the edges of the river and protected by up-stream braced pile bents and sheeting to form a shear structure, according to the revised plan.

The authors have written a very clear and complete description of an ingeniously designed bridge-repair project completed under difficult river and weather conditions.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

HYDROLOGY OF THE GREAT LAKES
A SYMPOSIUM

Discussion

BY MESSRS. G. H. HICKOX, C. R. PETTIS, AND HAROLD C. HICKMAN

G. H. HICKOX,[25] ASSOC. M. AM. C. E. (by letter).[25a]—The writer has read with interest Mr. Hickman's contribution to the subject of evaporation and feels that he is to be commended for bringing it to public attention. The fact that he is able to reach conclusions which vary widely from those held by others indicates that the subject is not thoroughly understood throughout the profession. Before his conclusions can be accepted, however, they should be examined carefully.

His principal conclusion, and the one which seems to be at greatest variance with generally accepted results, is with respect to the effect of vapor-pressure difference and wind on the evaporation rate. The paper states that the evaporation rate is not proportional to the vapor-pressure difference and that "the effect of the wind on evaporation is fundamentally different from that indicated by the common evaporation formula." The paper indicates that for one combination of water and air temperatures an evaporation rate of 0.09 in. per day might occur with zero vapor-pressure difference, the evaporation being due solely to the effect of wind at 10 miles per hr. It is entirely possible that water may be removed from a free water surface by wind when the vapor-pressure difference is zero. Such removal could be accomplished mechanically, the water being carried away from the evaporation pan or reservoir surface in the form of spray. This might easily occur when waves break, for example, the resulting spray being carried away by the wind on the form of minute droplets, without evaporation. There is evidence that such action occurs along the Pacific coast where the accumulation of salt on transmission line insulators several miles inland has become a serious problem. The writer is unable to conceive, however, of 0.09 in. per day being removed bodily from the

NOTE.—This Symposium was published in April, 1939, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: June, 1939, by A. A. Young, Assoc. M. Am. Soc. C. E.; September, 1939, by Messrs. Alfred J. Cooper, Jr., and Adolph F. Meyer; and October, 1939, by Messrs. S. T. Harding, O. E. Meinzer, and R. W. Davenport.

[25] Senior Hydr. Engr., TVA, Hydraulic Laboratory, Norris, Tenn.

[25a] Received by the Secretary December 1, 1939.

surface of Lake Superior by this means. Atmospheric turbulence would seem to be the only means of holding the droplets in suspension, and any lessening of wind velocity would result in a noticeable precipitation, it being assumed, of course, that evaporation could not occur.

Such speculation seems rather idle since the proportionality of evaporation to vapor-pressure difference has been well established. Dalton's Law has been accepted as a basis for correlating evaporation experiments for many years and is apparently sound theoretically. It has also been verified experimentally by Carl Rohwer,[26] F. Graham Millar,[27] B. C. Shepherd, C. Hadlock, R. C. Brewer,[28] and others. The three sets of experiments specifically referred to were conducted under the conditions suggested by Mr. Hickman; that is, "in a wind tunnel under laboratory conditions." The results were in agreement that, for any given wind velocity, the evaporation rate is proportional to the vapor-pressure difference. In these experiments, the vapor pressures were defined in the customary manner—as the pressure of saturated water vapor at the temperature of the water surface, and as the partial pressure of the water vapor in the atmosphere at a considerable distance from the water surface. C. W. Thornthwaite and Benjamin Holzman[29] have computed evaporation rates on the basis of Dalton's Law from observations of vapor pressures at different elevations above the ground surface. They have been successful not only in calculating evaporation rates when a positive vapor-pressure difference existed, but also in calculating the rate of condensation that occurred with a negative vapor-pressure difference.

No theory is any stronger than the evidence that supports it. Before Mr. Hickman's conclusions can be accepted, it will be necessary to refute the contrary results obtained in many carefully conducted experiments, or to explain them in terms of the new theory. It is believed that Mr. Hickman's conclusion regarding evaporation into a wind at a zero vapor pressure can be explained in terms of the accepted theory. His conclusion is apparently based on the improper method of using one daily observation of relative humidity to determine an average vapor pressure for the day. The variation of relative humidity in the vicinity of the Great Lakes is not known, but the point may be illustrated by reference to conditions in the Tennessee Valley. Figs. 12(*a*) and 12(*b*) show a typical example of the variation of relative humidity and air temperature throughout the day at a point in the vicinity of Norris Lake. The corresponding vapor pressure is shown in Fig. 12(*c*). It will be noted that in spite of considerable variation in temperature and relative humidity, the vapor pressure is relatively constant. In general, the actual change of the water vapor content of the air proceeds slowly. At night, when air temperatures are low, the water vapor content may be near saturation. During the day, when the temperature is high, the relative humidity necessarily drops, although the total water vapor

[26] "Evaporation from Free Water Surfaces," by Carl Rohwer, Assoc. M. Am. Soc. C. E., U. S. Department of Agriculture, *Bulletin No. 271*, 1931.

[27] "Evaporation from Free Water Surfaces," by F. Graham Millar, *Canadian Meteorological Memoirs*, Vol. 1, 1937, pp. 43 to 65.

[28] "Drying Materials in Trays," by B. C. Shepherd, C. Hadlock, and R. C. Brewer, *Industrial and Engineering Chemistry*, April, 1938, pp. 388 to 397.

[29] "The Determination of Evaporation from Land and Water Surfaces," by C. W. Thornthwaite and Benjamin Holzman, *Monthly Weather Review*, January, 1939, pp. 4 to 11.

(and hence its pressure) may not change greatly. The errors involved in applying a single relative humidity determination to the average air temperature in order to calculate an average vapor pressure for the day are obvious. In the case illustrated, the average air temperature is 63.5° F. Mr. Hickman does not state when the relative humidity was determined, but according to the source material[10] it was 9:00 a.m. Using this time for the data of Fig. 12, the relative humidity is 52% and the vapor pressure for a temperature of 63.5° is 0.77 cm of mercury. The actual average vapor pressure is 0.87 cm of mercury.

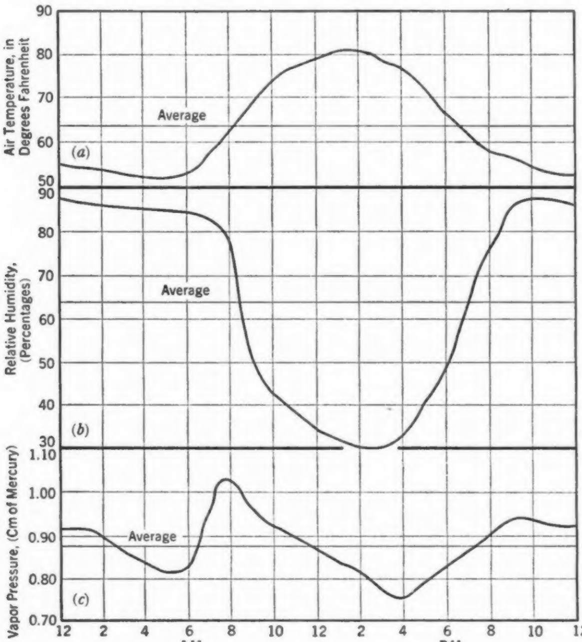

FIG. 12.—TYPICAL AIR TEMPERATURE, RELATIVE HUMIDITY, AND VAPOR PRESSURE NEAR NORRIS LAKE

Even the use of average air temperature and average relative humidity does not give the average vapor pressure, due to the non-linear relationship between vapor pressure and temperature. Referring to Fig. 12 again, the average air temperature of 63.5° F and average relative humidity of 64% give an apparent average vapor pressure of 0.95 cm of mercury, as compared with the true average of 0.87 cm. Similar reasoning applies to the use of average water temperatures to determine the vapor pressure at the water surface.

Dalton's Law is strictly applicable only when the true values of vapor pressure are used. The success of Meyer's formula (Equation (4)) is due to the empirical evaluation of the constants which takes care of the manner in which V and v are calculated. An equation containing constants derived in this man-

[10] A copy of the thesis, containing all observational data, has been filed for reference in the Engineering Societies Library, 33 W. 39th Street, New York, N. Y.

ner may give absurd results when applied outside the range of conditions which were used in its derivation. This would not disturb the validity of the fundamental laws on which it was based, and this is where Mr. Hickman "goes astray." It is quite possible to have a definite evaporation rate when the apparent vapor-pressure difference, determined from average air and water temperatures and one observation of relative humidity, is zero. It does not follow, however, that Dalton's Law is thereby proved to be in error.

The foregoing constitutes the chief objection to Mr. Hickman's thesis. The writer disagrees on a number of minor points which will be enumerated in the following paragraphs.

The assumption that "the evaporation from the open lakes, for a given air temperature, water temperature, and wind velocity, would be the same as the observed evaporation from the experimental pans for the same combination of air temperature, water temperature, and wind velocity" (see "Synopsis") is open to serious question. The data of the late Reuben B. Sleight,[30] Assoc. M. Am. Soc. C. E., and Mr. Rohwer[14] indicate that the rate of evaporation is reduced with an increase in the diameter of the evaporating water surface for diameters up to 12 ft. It does not follow that data from a pan 4 ft in diameter can be applied directly to a large lake without correction. This is to be expected from a consideration of the edge effects. Dry air passing over a water surface will take up water vapor more rapidly at the point where it first passes over the water than at some distance farther on. For this reason, the rate of evaporation from a small pan will be greater than that from a larger area.

Mr. Hickman states (see "Synopsis") that "The rate of evaporation from open-water surfaces when the air temperature is below freezing has been investigated for the first time." This statement overlooks Mr. Rohwer's work,[26] published in 1931, in which experiments made under these conditions were described. Mr. Millar's experiments[27] also apparently cover this range.

A comparison of the data obtained in these experiments with the records obtained by the Weather Bureau with their standard evaporation equipment would have been greatly facilitated if the same type of equipment had been used. The standard Weather Bureau pan is so mounted as to be exposed fully to sunshine and wind. The desirability of this exposure is open to criticism; but the provision of a water jacket or an insulated shell interferes materially with the similarity of the installation by reducing the transfer of heat through the walls of the pan both by solar radiation and by convection.

The writer is unable to see the reason for omitting relative humidity or vapor pressure as a basis of correlating the experimental results. Would it not be as logical to assume that September evaporation records would be grouped around a certain average value as to assume that any particular daily temperature in September should be accompanied by a certain relative humidity? Certainly it would make the work much easier if all the September evaporation

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[30] Discussion by the late Reuben B. Sleight, Assoc. M. Am. Soc. C. E., of paper "Evaporation on United States Reclamation Projects," by Ivan E. Houk, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 90 (1927), pp. 303 to 316.

[14] "Evaporation from Different Types of Pans," by Carl Rohwer, Assoc. M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 99 (1934), p. 673.

records could be grouped and averaged. Such a value would probably be satisfactory for average conditions.

Correlation between the evaporation rate and relative humidity as such is not to be expected in any case. The proper basis for correlation, as predicted from physical theory and verified by experiment, is between the evaporation rate and the difference of vapor pressure. Relative humidity, as implied by the name, is purely a relative term and has no absolute value.

Mr. Hickman's expressed desire for tests made in a wind tunnel under laboratory conditions has already been fulfilled by experiments made by workers in various fields, including the Canadian Meteorological Service, which is also studying the hydrology of the Great Lakes. It is unfortunate that the results of these experiments are not more generally known to the civil engineering profession.

C. R. Pettis,[31] M. Am. Soc. C. E. (by letter).[31a]—Since 1936 the writer has searched for quantitative information about underground water. Harold Conkling, M. Am. Soc. C. E., estimated that 40% or 50% of the rainfall on Long Island reaches the water table; and he stated that probably most of the underground water reaches the sea without coming to the surface.[32] A few engineers in California seem to realize that, at times, ground-water movement may involve appreciable quantities; but engineers in general seem to be almost unanimously of the opinion that ground-water movements are negligible, and can be neglected entirely in the study of the hydrology of any region. Messrs. Harding, Meinzer, and Davenport state the generally accepted opinion clearly and emphatically. Mr. Freeman[7] and Robert E. Horton,[2] M. Am. Soc. C. E., started their studies on the hydrology of the Great Lakes with the assumption that ground-water flow is negligible, and the assumption is carried over into their conclusions. From comments by Mr. Harding it seems that the same was probably done in the study of Pyramid Lake and Walker Lake.

The writer tried to outline the method so that it could be followed by any one who wished to make a similar study; and, in the paper, he gave all of his conclusions that seemed to be of general interest. The statistical studies upon which the conclusions were based[5] were voluminous, and it was not thought that they would be of sufficient general interest to justify publication.

The writer attempted to reach his solution without being unduly influenced by preconceived opinions; but he recognizes that it is fair and logical to ask the question, "Is the value of U too high?" A few remarks will be offered on this question. Necessarily, most of the evidence is circumstantial, but it is cumulative, and indicates that the underground flow, U, is appreciable, and that its numerical value is not much less than the value of the surface flow R.

The underground flow into the lake in the period from April 1 to July 31, inclusive, is greater than the evaporation from the lake in the same period of

[31] Colonel, Corps of Engrs., U. S. Army (*Retired*), Head, Dept. of Math., Mississippi State Coll., State College, Miss.

[31a] Received by the Secretary November 17, 1939.

[32] *Transactions*, Am. Soc. C. E., Vol. 102 (1937), p. 759.

[7] "Regulation of the Great Lakes," by J. R. Freeman, p. 421.

[2] National Research Council Report, 1936, p. 351.

[5] A copy of the thesis, including all observations and other data, has been placed on file in the Engineering Societies Library, 33 West 39th Street, New York, N. Y.

time. The evidence to support this statement is direct, and is based on only one geometrical assumption—namely, that water which appears in the lake, but which neither falls on its surface nor flows into it from surface streams, must be classed as underground flow. From Fig. 1(*c*), it may be noted that the average value of $(E - U)$ for the period April 1 to July 31, inclusive, is negative (-0.2). This means that for the stated 4-month period the underground flow is greater numerically than the evaporation from the lake surface, both being measured in land units. In other words, if there is any net evaporation from the lake in the average period from April 1 to July 31, there must have been at least an equal quantity of underground flow coming into the lake in the same period, in order to make the water accounts balance.

It is believed that the data may be considered to be excellent and accurate for the purpose of the foregoing statements, and that the statements are true without appreciable error.

The solution in the text indicates that the evaporation from the lake, in lake units, from April 1 to July 31, inclusive, is 6.4 in.; and the underground flow for the same period is 6.6 in., in lake units. This value of evaporation for the stated period checks very well with the results obtained by Mr. Hickman in his analysis of the evaporation experiments.

If an effort is made to check the values of evaporation from standard text-books, difficulty is encountered. The evaporation experiments which have been made in this latitude[12] (and which have been mentioned elsewhere by Mr. Meyer) indicate an evaporation from April 1 to July 31 (which would mean a corresponding underground flow) almost three times the values given in the preceding paragraph. Mr. Meyer there[33] indicates about 80% more evaporation than in Fig. 1, and Mr. Meyer's chart indicates at another place[34] about 36% more evaporation. The writer's values for evaporation and underground flow for the stated period are the smallest that are consistent with any known published data on the subject.

An abnormally high rainfall, B, in any one month will ordinarily increase the surface run-off R in the same month, and to some extent in the following months; the effect of the high rainfall on R can be computed directly from the data. The effect of a high rainfall on the underground flow U can likewise be computed by a method which will be described. The relative effects of the high rainfall on R and U can be compared therefore.

An abnormally high rainfall in any given month has no appreciable effect on the value of $(U - E)$ for the given month; but there is a strong tendency for the value of $(U - E)$ to be above normal for both the second month and the third month, and this tendency is quite pronounced not only on Lake Superior, but on each of the other Great Lakes.

An analysis of the data with respect to variations in evaporation E indicates that temperature is the main factor affecting E. Similarly it is evident that rainfall B is the principal factor affecting underground flow U. Furthermore, over longer periods of time, such as one year, there is no apparent corre-

[12] "Elements of Hydrology," by A. F. Meyer, John Wiley & Sons, Inc., New York, N. Y., 1917.
[33] *Loc. cit.*, p. 235.
[34] *Loc. cit.*, p. 236.

lation between temperature and rainfall; U and E are independent variables. If a group of values of $(U - E)$ are selected where there was high rainfall in some preceding month, and a similar group following low rainfall in the same preceding month, the difference in values of $(U - E)$ will tend to represent the difference in values of U, since the values of E, being independent, will tend to cancel. The greater the number of items, the more accurate the results from the assumption will be. This method has been used with many different combinations, and the indication is always the same—that high underground flow tends to follow high rainfall, nearly all of the effect being in succeeding months.

To illustrate, consider the month of September for the 15-yr period 1920 to 1934. For the years 1921, 1926, 1928, 1929, 1930, 1931, and 1933, the average value of B for September was 4.3 in., the average value of R for October was 0.8 in., and the average value of $(U - E)$ for October was (-1.5) in. For the years 1920, 1922, 1923, 1924, 1925, 1927, and 1932, the average value of B for September was 2.7 in., the average value of R for October was 0.7 in., and the average value of $(U - E)$ for October was (-1.9) in. Subtracting the different items, it appears that in the first period an average monthly excess of 1.6 in. of rainfall, B, produced an increase of 0.1 in. in surface run-off R in October, and an increase of 0.4 in. in underground flow U, for October. Since the value of U was computed for seven values of $(U - E)$ in years of high rainfall, and seven values in years of low rainfall, it is possible that the value of U contains some slight error because the values of E have not balanced. However, if the foregoing process is repeated a number of times, and the results combined for a final conclusion, the probability is that the final results will contain no appreciable error.

The foregoing method was used for various periods of months, and throughout the 15-yr period. For example, high and low rainfall for corresponding periods were compared, for periods varying from one month to twelve months, and the indicated effects on underground flow U were studied for the following months, up to twelve months. The indication is definite and consistent, that underground flow to the Lake increases after high rainfalls. This indication is clear in all of the Great Lakes, for the 15-yr period. It seems that the peak of underground flow into the Lake follows the high rainfall by about six weeks. It then decreases. The study indicated that the effect of a high rainfall on underground flow will not vanish completely for several months.

One calendar month was the unit of time used for all the basic data. Fifteen years was the total period considered for the four Great Lakes. The number of months considered was 720, or 180 months for each Lake. Considering the statistical methods used, the writer is of the opinion that his results cannot be considered to be absolutely accurate; but that they are mathematically excellent approximations, and the error of current engineering opinion about ground-water has been proved beyond a doubt.

In the Freeman report[35] certain reasons are given for believing that the actual rainfall may be greater than that indicated by U. S. Weather Bureau records. More than 100 stations were considered in the Lake Superior area,

[35] "Regulation of the Great Lakes," by John R. Freeman, 1926.

and most of them are not operated by Weather Bureau personnel. It is believed that most of the errors due to location and other causes would tend to decrease the quantity of rain reported. The writer has seen a rain gage that was dry after a light summer shower. The gage was warm, and the water evaporated almost as soon as it fell. The writer is of the opinion that the actual annual rainfall may have been 3 in. more than the value that was used in his study. The effect of this on the computations would be to add 3 in. to the land losses, without affecting the underground flow.

Mr. Horton has published a summary[36] of engineering opinion about land losses. It is thought that the writer's conclusions about land losses are not inconsistent with Mr. Horton's statements and tables, if local conditions are considered; although the writer's data are slightly on the low side.

The area of Lake Nipigon, on the north shore of Lake Superior, estimated from a small-scale map, is about 1,600 sq miles. All other lakes in the Lake Superior drainage area are relatively small. It is thought that the water surface in the land-drainage area is not more than about 5 per cent. As far as is known, Lake Nipigon and the other lakes freeze in the winter and remain frozen about five months, so that evaporation from them is necessarily much less than from Lake Superior. The writer's data apply to Lake Superior, and they will not apply to localities where conditions are different.

The entire Great Lakes region was covered by glacial drift. Considerable old beach sand is in evidence in places. The writer has made numerous automobile trips covering areas around the Great Lakes; he has talked to geologists and mine operators; and he has found nothing to indicate the impossibility of a large underground flow.

The writer does not claim that he has said the final word, even about Lake Superior. He only hopes that he has offered evidence about a subject on which evidence is needed.

Correction for *Transactions:* In *Proceedings* for September, 1939, page 1311, line 23 should read: " * * * lying in a belt extending from half-way between the 3-in. and 4-in. lines to half-way between the 4-in. and 5-in.-per-month * * *."

HAROLD C. HICKMAN,[37] JUN. AM. SOC. C. E. (by letter).[37a]—During the early stages of the investigation of the hydrology of the Great Lakes certain unexpected relations of negative yield became apparent, which, after making due allowances for errors of observations, could be explained only by unexpectedly large evaporation in winter and correspondingly smaller evaporation in summer. Mr. Freeman had given careful consideration to all available data on evaporation in his study of the hydrology of the Great Lakes, and advised further study of the subject.

Before experiments were begun the writer reviewed most of the work that had been done on evaporation to familiarize himself with conditions under which this work had been conducted, to determine if these conditions were comparable

[36] "Hydrology of the Great Lakes," by Robert E. Horton.

[37] Junior Engr., U. S. Lake Survey, Detroit, Mich.

[37a] Received by the Secretary December 11, 1939.

to those in the Great Lakes, and also to determine whether the results of this work could be used in arriving at a suitable and accurate value of evaporation from the lake surface. It was found that conditions affecting evaporation from the lake were quite different from those under which all available accurate data on evaporation from other large bodies of water had been obtained, and very different from the conditions around the small experimental tanks from which formulas and constants for estimating evaporation had been derived.

The areas of the water surfaces of the lakes are so large, and the distances across in the direction of the wind so great, that factors affecting evaporation under these conditions must be considered separately. The comparatively mild winters, cold damp springs, cool mild summers, and warm prolonged autumns, characteristic of the lake region, are evidence of the influence of the broad, deep expanse of these waters on climatic conditions. The climate and wind movement, the amount of moisture in the air (which largely affects both radiation loss by night and absorption of heat during the day), the depth of the water which tends to keep the water temperature (as in the case of Lake Superior) between 33° F in winter and 51° F in summer—all are factors in a particular evaporation problem.

The experiments reported in the original thesis[10] were not undertaken to present a new formula, nor to detract in any way whatsoever from the work that had been done previously in this field. They were conducted to determine the best available value for monthly evaporation from the surface of the Great Lakes, using only data which the writer considered as accurate and as having a direct bearing on the rate of evaporation, and excluding from consideration data over which controversy might arise. In the discussions certain objections were raised to the methods used in arriving at a monthly value of evaporation based on water temperature, air temperature, and wind velocity, to the exclusion of relative humidity. Mr. Cooper presents data which show that, apparently, evaporation in the Tennessee Valley was influenced by both relative humidity and wind velocity, but with greater effect being due to relative humidity. He also states that, in his opinion, any revision of current opinion as to the effect of wind on evaporation should be toward giving less weight to wind and more to relative humidity. The writer spent considerable time studying meteorological conditions in the locality in which Mr. Cooper's experiments were conducted and found that, in no degree, did they approach conditions existing in the lake region.

The Tennessee stations were equipped in accordance with U. S. Weather Bureau "Class A" evaporation specifications. The principal objection to this type of installation lies in the rapid changes of pan temperatures. The water temperature tends to follow that of the air and for the purpose of an investigation of evaporation from large cold bodies of water is entirely unsatisfactory. Data obtained with equipment of this type do not make allowance for the large differences between water temperature and air temperature so characteristic in the lake region. It is quite conceivable that the Tennessee experiments would show that evaporation is influenced more by changes in relative humidity than wind velocity when the equipment used and the conditions under which the experiments were conducted are considered. An air mass passing over a body

of water at approximately the same temperature as the air will absorb water vapor in quantities depending upon the saturation deficit of that air. At low wind velocities it is evident that the quantity of water evaporated under these conditions would be influenced more by relative humidity than by the velocity of the wind.

In the lake region the air temperature is influenced by the temperature of the lake water at all times. The temperature on shore may be 80° and 5 miles out in the lake it may be 60°. During a large part of the year "thermal" winds blow off the lake. In the winter months a cold dry mass of air, passing across a lake that does not freeze over, tends to "warm up" and consequently the carrying capacity is increased. Under these conditions evaporation would go on during the winter months contrary to all preconceived ideas. That this carrying capacity does exist in winter is evident in the snow belt on the south shore of Lake Superior and Lake Ontario. It is likely that evaporation under these conditions is influenced more by the wind than by relative humidity, as reported by the Weather Bureau Stations around the lakes, and that these relative humidity data cannot be used as a means of determining evaporation from the Great Lakes.

Mr. Cooper states that a few relative humidity readings during the day will not give the average value for that day except in isolated instances. It will be noted that the writer stated clearly that no evidence could be found wherein the amount of evaporation varied with either the relative humidity as read at the station or as determined at Weather Bureau Stations in the vicinity. Mr. Cooper suggests that, because the daily average value was difficult to obtain, it was neglected. It might be well to point out that tests to determine the effect of this factor were conducted extensively before actual experiments began. The average value for the day was obtained by averaging the hourly readings throughout each 24-hr period. Although a number of daily tests were made, four typical separate days may be noted to show the effect of humidity. The first set of two days had an average water temperature for the day of 52° and an average air temperature of 50°. The second set had an average daily water temperature of 56° and an air temperature of 64°. The daily wind velocity for the four days was 5 miles per hr. The evaporation for the days, under the first set, was 0.124 and 0.128 in. with average daily relative humidities of 67 and 85 per cent. The evaporation for the second set was 0.058 and 0.055 in. with respective relative humidities of 73 and 81 per cent. In a number of other instances evaporation continued during periods of complete saturation under a strong wind. Evidence of this kind was found throughout the study and it seems, in the light of these facts, that relative humidity is not as important as hydrologists have generally believed.

The data presented by Mr. Cooper are confined to air temperatures between 40° and 46° and water temperatures between 44° and 50° with wind velocities ranging from 0.5 to 1.5 miles per hr. It would be interesting to see what results could be obtained by including the data from a number of stations with air temperatures ranging from 10° to 95° and with water temperatures from 33° to 75° with wind velocity up to 22 miles per hr.

Although evaporation was found not to be related to relative humidity as reported at the U. S. Weather Bureau Stations, these stations, when the average monthly values were compared from year to year, did show a certain degree of consistency. The average monthly relative humidity for the years from 1920 to 1934 for the Superior Basin was computed by averaging the monthly means as reported by the Weather Bureau Stations at Sault Ste. Marie, Mich.; Marquette, Mich.; Houghton, Mich.; Duluth, Minn.; and Port Arthur, Ont., Canada.

Table 9 shows the averages of relative humidity for the Superior Basin. Although these data should not be construed as representing open lake conditions they do indicate what changes occur from season to season. Note the regularity of descent from a high in January to a low in May and the progressive

TABLE 9.—RELATIVE HUMIDITY, LAKE SUPERIOR

Year	January	February	March	April	May	June	July	August	September	October	November	December	Mean
1920	92	86	82	78	64	76	74	76	78	78	85	92	80
1921	91	90	85	72	72	75	76	79	79	81	86	87	81
1922	89	87	82	80	76	75	76	78	81	80	86	86	81
1923	87	84	84	73	66	72	76	77	82	76	80	83	78
1924	87	85	80	74	62	70	76	80	83	82	82	86	79
1925	86	86	79	64	64	75	77	78	82	80	81	86	78
1926	87	86	82	70	66	70	76	80	86	81	85	89	80
1927	87	85	79	71	76	73	77	75	79	79	86	88	80
1928	85	85	78	73	66	74	81	83	84	84	78	83	80
1929	85	86	82	72	68	71	72	73	77	76	80	88	77
1930	86	88	81	71	71	72	77	64	77	84	81	88	79
1931	87	84	82	70	72	76	74	74	81	79	80	82	78
1932	85	85	82	69	65	74	74	78	75	80	83	86	78
1933	84	86	82	72	68	69	73	70	74	80	86	89	77
1934	89	87	82	80	62	72	70	70	83	..	..	..	..
Average	87	86	81	73	68	73	75	76	80	80	83	87	79

increase back to a high in December. If relative humidity does influence evaporation it must vary directly with evaporation in the lake regions as indicated in Table 9 and inversely as shown by Fig. 8 in Mr. Cooper's discussion of the Tennessee Valley.

It is believed that a seasonal variation in relative humidity, which is rather uniform from year to year, does exist in the lake region. It does not follow, however, that because of this fact evaporation is the same from year to year.

The objections stated by Mr. Meyer were confined to the data upon which the final conclusions were based. He states that no records were on file[10] which even reasonably simulated conditions prevailing on Lake Superior from March to August and proceeds to show that for the summer months of April to September the average daily water temperatures in the pan were from 6.6° to 21.1° higher than those shown by the open lake water temperatures curve and upon which the values of monthly evaporation in the Pettis analysis were based.

The paper describes clearly the procedure used in analyzing the data from the various stations and, as stated, these data are on file in the Engineering Societies Library. The thesis states that[10]

> "one group of 44 readings—11 from Duluth, 14 from Kewaunee, 10 from Detroit, and 9 from Buffalo,—where the average daily air temperature of each reading was between 40° and 45° and where the average daily water temperature was between 45° and 50°, were included in this group. The centroid of these 44 readings was found and plotted, and the average of these readings assigned this point. The average wind velocity of the group was found to be five miles per hour."

When a method of analyzing data of this kind is used, any departure of one station from the average water temperature curve for any one month does not exclude the station entirely from consideration.

In the original analysis, 185 observations, a good portion of which were obtained from the Kewaunee Station, were used, where the water temperatures ranged from 35° to 40°, and air temperatures from 10° to 45°; 193 readings with the water from 40° to 45° and the air from 10° to 50°; 151 readings where the water temperature was between 45° and 50° and the air from 20° to 50°; and 100 readings ranging from 50° to 55° for the water and 35° to 70° for the air. It is believed that this range of air and water temperature does simulate conditions prevailing on Lake Superior.

It must be admitted that the control of the pan water temperature was not all that could be desired. It was extremely difficult to obtain this control, particularly during the summer months, as funds for mechanical refrigeration were not available. The various stations had to depend upon cold well water which did not provide sufficient cooling effect except at Kewaunee. The experiments have been continued with better efficiency and the indications are that the ever-increasing number of daily observations will strengthen the curves materially and add to their range.

It is believed that the analyses of these experiments in conjunction with the air-temperature and water-temperature curves in the paper do give a much more satisfactory value for monthly evaporation from the Great Lakes than any other method or formula presented heretofore.

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DISCUSSIONS

DESIGN OF AN OPEN-CHANNEL CONTROL SECTION

Discussion

By Messrs. George E. Barnes, and Karl R. Kennison

George E. Barnes,[11] M. Am. Soc. C. E. (by letter).[11a]—On the subject of open-channel control sections, the author is to be credited with: (1) Definition of the problem in general terms amenable to analysis, (2) development of the topic by several original steps, and (3) valid conclusions and results, including certain derived equations of useful form.

Unfortunately, the validity of the author's derived equations is not established by the methods followed in the paper. Equations (7), (10), (14), and (15) depend on Equation (6). The writer challenges the author to defend his derivation of Equation (6) on the basis of the reasoning given in the paper—namely, "Since Q is a maximum, $\frac{dQ}{dA_c}$ can be equated to 0 so that the relation between Q and A_c is $Q = A_c^2 \, g \, m$."

Relative to the foregoing, the following comments are pertinent: If, as the author states, Q is made to vary under constant H_p, the resulting curve (see Fig. 5) is of such form that Q becomes a maximum at some particular value of d, designated as the critical depth. Since $\frac{dQ}{dd}$ or $\frac{dQ}{dA}$ is known to be continuous and is known to change in sign from plus to minus at the point of critical depth, the slope of the tangent to the curve at that point is zero. Therefore, the numerator of the right-hand side of Equation (5) is equal to zero when the denominator is not zero. However, if both numerator and denominator are zero at d_c, further investigation of the nature of Q as a function of A_c is necessary. The author's assumption that the denominator is equal to zero cannot be accepted without adequate proof. Lacking such proof, the correctness of the succeeding equations is open to question.

Note.—This paper by Karl R. Kennison, M. Am. Soc. C. E., was published in May, 1939, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: September, 1939, by Messrs. V. L. Streeter, Emery H. Willes, and Robert O. Thomas; and October, 1939, by Frank S. Bailey, Assoc. M. Am. Soc. C. E.

[11] Prof., Hydr. and San. Eng., and Head, Dept. of Civ. Eng., Case School of Applied Science, Cleveland, Ohio.

[11a] Received by the Secretary November 22, 1939.

The method used by the writer to determine the author's equations is as follows: In any open channel of whatever cross-section, the elevation of the energy gradient above the channel bed is $H = h_v + d$ (see Fig. 6). For any

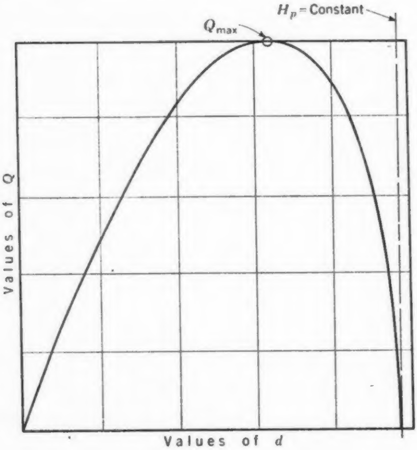

FIG. 5

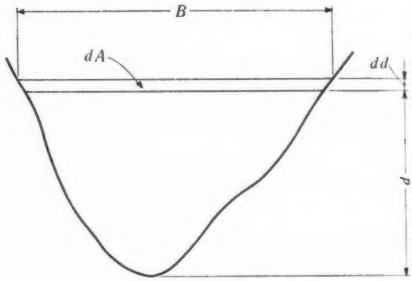

FIG. 6

particular value of Q the channel proportions for which H is a minimum (flow at critical depth) are found thus:

$$H = \frac{Q^2}{2\,g\,A^2} + d \qquad (50)$$

$$\frac{dH}{dd} = -\frac{Q^2}{g\,A^3} \times \frac{dA}{dd} + 1 = 0 \qquad (51)$$

but $\frac{dA}{dd} = B$; hence

$$Q^2 = \frac{g\,A_c^3}{B} \qquad (52a)$$

and

$$\frac{Q^2}{2\,g\,A_c^2} = \frac{A_c}{2\,B} \qquad (52b)$$

Referring to Fig. 1—by construction

$$d = H - \frac{Q^2}{2\,g\,A^2} = m'\,Q + m'\,n' - \frac{Q^2}{2\,g\,A^2} \qquad (53a)$$

By definition, the slope of the rating curve is

$$S = \frac{dd}{dQ} \qquad (53b)$$

and, by definition, the slope of the depth versus energy-head curve is

$$m' = \frac{dH}{dQ} \qquad (53c)$$

Then, differentiating Equation (53*a*),

$$\frac{dd}{dQ} = \frac{dH}{dQ} - \frac{Q}{g\,A^2} + \frac{Q^2}{g\,A^3} \times \frac{dA}{dQ} = \frac{dH}{dQ} - \frac{Q}{g\,A^2} + \frac{Q^2}{g\,A^3} \times \frac{dA}{dd} \times \frac{dd}{dQ} \quad (54)$$

and, by substitution $\left(\frac{Q^2}{g\,A_c^3} = \frac{1}{B}\right)$:

$$S = m - \frac{Q}{g\,A_c^2} + \frac{1}{B} \times B \times S \quad (55)$$

Hence, $m = \frac{Q}{g\,A_c^2}$, which establishes the validity of Equation (6). The writer has checked and concurs with the subsequent derivations leading to the working formulas, Equations (7), (10), (14), and (15).

A check is secured on the author's computations for the trapezoidal section, at peak discharge, as follows: The required value of $A_c = \sqrt[3]{\frac{28.3^2 \times 1.092}{32.2}} = 3.01$. If the control section is plotted to scale, the planimetered value of A_c, as drawn, will be found to agree. It follows also that the computed value of $h_{vc} + d_c = H_p$ (Equation (1*c*)), which proves to be the case, provided that the reference plane for both control and piezometer sections is taken at the bottom of the piezometer section for the values shown in Table 1.

The writer would also like to comment on certain matters touched upon in the paper, bearing on applications of the method and practical considerations.

A control section is one at which the minimum energy head for any discharge is independent of the energy head down stream. The particular control section to be adopted for any given case depends first on the function it is to perform. One common function of a control section is the measurement of discharge; another, perhaps less common, is that of regulating velocities up stream. With intelligent design, it is possible to combine the two functions in one structure, although the requirements for one do not necessarily meet the requirements for the other. In the matter of measurement, sensitivity of the water-surface level with respect to discharge is important. In the matter of velocity regulation, not only is the average velocity up stream of importance, but the effect of the control section on velocity distribution up stream may be important as well. Frequently, the necessity of conserving head has considerable influence on the selection of a suitable control section. The author's statement that his method "is claimed * * * to have possibilities in the application to practical problems" is a modest one. The writer believes that the author's paper, if read with understanding, will do much to offset a tendency, sometimes noted, toward the adoption of accepted types of control sections, after only superficial investigation, in situations where an original design is much to be preferred.

In the general case, when flow is at critical depth, the velocity head is half the average depth (see Equation (52*b*)), and the energy head at the control section is 1.5 d_{average}. Of course, this is H_p minus losses. Where both the control section and the up-stream sections are of regular geometric shape, the equation, $H_p = H_c$ + losses, expressed in terms of the variables including d_c, may be set up without great difficulty. However, except for the very simplest

cases, its solution may be too complicated for handling except by "cut-and-try" methods. The writer has had occasion to design control sections for grit chambers for the purpose of measuring discharge, or for regulating velocities, or both (see Fig. 7). In such cases, the velocity head in the grit chamber is so small as to be practically negligible, and the difference in water-surface elevation between the control section and the up-stream section may be taken as the

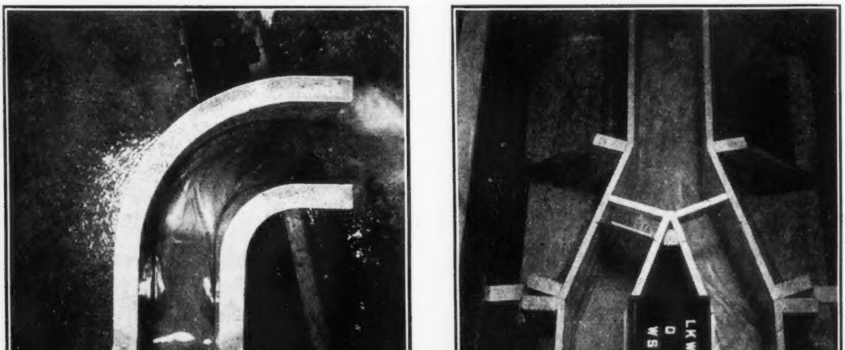

FIG. 7.—FLOW IN CONTROL SECTIONS DESIGNED FOR GRIT CHAMBERS (MODEL SCALE 1 : 6)—(a) WARD'S ISLAND, NEW YORK, N. Y. (FLOW EQUIVALENT TO 86 CU FT PER SEC); (b) LAKEWOOD, OHIO (FLOW EQUIVALENT TO 26 CU FT PER SEC)

velocity head at critical depth in the control section, plus the head loss in the transition. The head loss in the transition may be quite closely estimated, or, if necessary, determined by experiment, as a function of this velocity head. The following simplified equation may then be written:

$$Z_g = Z_c + d_c + (1 + k)\, h_{vc} \quad \ldots\ldots\ldots\ldots\ldots\ldots (56)$$

in which Z_g = the water-surface elevation in the grit chamber, and Z_c = invert elevation in the control section. It is important to note in this connection that conditions may permit designing the up-stream section to fit a suitable control section, as an alternate to the more obvious procedure of designing the control section with respect to the up-stream section. By arbitrary proportioning of the control section and the up-stream section, suitable design for a given range of flows often may be secured with readiness.[12] Such cut-and-try methods, as stated by the author, may be just as useful and important as others.

[12] "Unique Solution for Grit Chamber Design," by George E. Barnes, *Waterworks and Sewerage*, September, 1939.

If the performance of the control section is to be measured, it will probably be with piezometers located either at the up-stream section or at the control section, or both. Piezometer readings are most likely to be accurate when taken in sections of undisturbed flow, at low velocities, and in the absence of standing waves or cross currents. Fig. 7 illustrates the occurrence of standing waves in the throat of two different control sections, and the comparatively smooth and undisturbed surface condition up stream. Thus there is likely to be better agreement between the hydraulic gradient and the piezometer reading when observations are taken at the up-stream section. Furthermore, for a given rating curve with Q varying from zero to some peak discharge, the range of depth in the up-stream section is greater than at the control section, and consequently the up-stream water-surface level is a more sensitive indicator than the control section water-surface level. In some cases, adherence to the belief that critical depth can be measured with piezometers at some point in the control section has resulted in the installation of open-channel measuring devices which are less sensitive than they might be with the piezometers located up stream. Throughout his paper, the author refers to the up-stream section as the piezometer section. This would seem to imply, although it is not specifically stated, that the location of piezometers in the up-stream section is desirable or to be expected. In any case, and for the reasons outlined previously herein, the writer would recommend strongly that, in the usual situation, piezometer readings be taken up stream rather than at the control section.

The writer is not quite clear as to the author's statement that the reason for raising the invert of the channel at the control section by some arbitrary amount is to offset any tendency toward surface waves. Of course, it is necessary to raise the invert, or otherwise to constrict the control section, to insure that there is a localized break in the hydraulic gradient. Otherwise the true location of the control section would not be known so accurately. The author does not point out other means for constricting the control section which are perhaps equally suitable. In the writer's experience, it has often been preferable to reduce the width of the control section all the way from floor level to free-board level with, or without, raising the invert.

The author limits the scope of his paper to a precise statement of the theory involved and a short discussion of some practical applications. Any further discussion by the writer of practical considerations would be beyond the limits of the paper as written. The paper is remarkably comprehensive, considering its brevity, and is written with an accurate conception of the practical aspects of the subject.

KARL R. KENNISON,[13] M. AM. SOC. C. E. (by letter).[13a]—There is a reasonable unanimity among the various discussions as to the correctness of the theory advanced and conclusions reached. It is evident that the writer's aversion to the publication, in *Proceedings*, of lengthy details of mathematical equations has gotten him into difficulties and led to the expenditure of much time and effort by those discussing the paper which might have been avoided.

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[13] Chf. Engr., Met. Dist. Water Supply Comm., Commonwealth of Massachusetts, Boston, Mass.

[13a] Received by the Secretary November 27, 1939.

Prior to its last-minute hasty revision, the paper contained the following explanation of the method of finding Equation (6), which should meet, satisfactorily, the criticism properly aimed at the explanation as published: Combining Equations (1) and (2), Equation (4), adapted to the condition of critical flow through the controlling section, becomes

$$\frac{Q^2}{2\,g\,A_c^2} = m\,Q + m\,n - d_c \qquad (57)$$

There are five variables in Equation (57), but m and n are constant for small or differential changes in the others and hence do not affect, other than as constants, the relation which it is desired to find between the other three. One of the conditions that makes the section a controlling or critical section is that, for any one value of the variable depth d_c and with the same relation between all the variables and this tentatively constant depth, the area A_c must be the minimum area that will satisfy the equation; because if any smaller area at such constant depth could discharge as much as the rating curve calls for, then A_c is too large. That is, some other smaller critical section must be holding the water back. Accordingly, for present purposes, one may: Tentatively consider m, n, and d_c as constants; differentiate Equation (57) with respect to the variables Q and A_c; and—since A_c is a minimum and $\frac{dA_c}{dQ}$ equal to 0—write:

$$\frac{dA_c}{dQ} = \frac{A_c}{Q} - \frac{A_c^3\,g\,m}{Q^2} = 0 \qquad (58)$$

Hence (see Equation (6)) $Q = A_c^2\,g\,m$. The independent proofs developed in the discussion, such as that of Mr. Willes and Mr. Barnes, are very gratifying.

Mr. Streeter questions why Equation (2) requires the condition of critical flow. This is because it involves m and n which, by definition, are tied to the rating curve of the controlling section. In other words, they apply to the condition of critical flow through the controlling section.

It is impossible for the writer to follow the reasoning of Mr. Thomas in his criticism that the use of such a controlling section precludes an economical approach channel. Whatever the maximum discharge of the economically designed channel is—whether 28.3 cu ft per sec or 43.5 cu ft per sec—this is, of course, what determines the maximum capacity of the controlling section, or the upper end of the desired rating curve. The slope of the rating curve must fit the conditions of available head. If there is a dearth of available head for the measurement, then the rating curve must be correspondingly flat. Within the range of desired flows there will be times when there is some non-uniform flow or back-water curve; but this should not affect the economical design of the main channel in any way, and any special modification of this channel that might be needed would be confined to a very short length immediately up stream from the piezometer section.

Answering some of the inquiries by Mr. Barnes: It is true, and should be stated definitely, that the location of the piezometers is in the up-stream section. If they were near the control section, recording would be erratic and unpredict-

able, because, when flow is nearly at the critical depth, the upper and lower alternative stages are so close together that the surface jumps easily from one to the other in waves. For the lower rates of flow, in the practical examples given for illustration, this tendency would extend up stream to the piezometer section if the invert at the control section were not raised to prevent it. The accurate recording of a water surface depends on forcing the alternative stages apart from each other. As Mr. Barnes points out the same thing could be done by reducing the width instead of raising the invert. However, the latter was better adapted to the examples given which sought a straight-line rating curve of maximum steepness.

In conclusion the author is gratified to note an appreciation of the comprehensive character of this study and of its adaptation to practical aspects.

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DISCUSSIONS

THE UNIT HYDROGRAPH PRINCIPLE APPLIED TO SMALL WATER-SHEDS

Discussion

BY FRANKLIN F. SNYDER, JUN. AM. SOC. C. E.

FRANKLIN F. SNYDER,[8] JUN. AM. SOC. C. E. (by letter).[8a]—The usefulness of the unit hydrograph principle in analyzing rainfall and run-off records from small water-sheds is well illustrated in this paper. The author fills the gap, with respect to size of area to which the procedure is applicable, from the city block as demonstrated by W. W. Horner, M. Am. Soc. C. E., and F. L. Flynt, Assoc. M. Am. Soc. C. E.,[9] to drainage areas of several hundred square miles, and larger, as demonstrated by other investigators. Although the paper is quite complete, the writer would like to discuss several items with the hope of increasing the value of the paper as a guide for further studies.

The author has established a procedure for separating ground-water and surface run-off and apparently has been able to ignore the occurrence (if any) of sub-surface storm flow. On larger basins the unit hydrograph crests for large floods are frequently higher than the crest values obtained for small floods, and this is due in part to the difficulties encountered in handling true surface run-off, sub-surface storm flow, and ground-water flow.

From the statements on the separation of surface run-off from ground-water, it appears that the author's procedure includes, with the ground-water discharge, most of the sub-surface storm flow which has entered the surface channels within the confines of the basin. If this could be done for floods and basins of all sizes, it would eliminate the need to consider sub-surface storm flow in connection with unit hydrographs. Since sub-surface storm flow would become effective at the gaging stations shortly after the beginning of run-off, the inclusion of the former with run-off from the ground-water zone would explain the author's conclusion that the rate of ground-water contribution increases as soon as run-off begins.

NOTE.—This paper by E. F. Brater, Jun. Am. Soc. C. E., was published in September, 1939, *Proceedings*. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion of the paper.

[8] With Federal-State Flood Forecasting Service, Commonwealth of Pennsylvania, U. S. Weather Bureau, and U. S. Geological Survey, Pittsburgh, Pa.

[8a] Received by the Secretary November 21, 1939.

[9] "Relation Between Rainfall and Run-Off from Small Urban Areas," by W. W. Horner and F. L. Flynt, *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 140.

The writer believes it possible that for a short period of time the ground-water discharge may actually decrease at a greater rate following the occurrence of run-off than before. At some time near the occurrence of crest discharge the ground-water discharge begins to increase and then probably follows a trend similar to that described in the paper. As shown by Mr. Brater, the actual transition would be smooth, regardless of its exact course. However, in practical applications, the separation can perhaps be made more consistent for application to different basins by the use of several straight lines rather than a curved line.

The author has concluded that the period of rise of a unit hydrograph is independent of the duration of rainfall so long as the duration of rainfall does not exceed the period of rise. This is contrary to results obtained from larger drainage basins, and there are several indications that it may be erroneous for the water-sheds under discussion.

There is usually a period at the beginning of rainfall during which no run-off occurs, and the rainfall in this interval is often referred to as initial loss. The duration of initial loss may vary considerably for different storms, especially in summer. The duration of initial loss should be subtracted from the duration of the storm to obtain the duration of the surface run-off producing rain. The latter period of time is the duration of rainfall that should be compared with the period of rise of the resulting discharge graph.

Although the writer's experience has indicated that values of initial loss are usually smaller in the Southern Appalachians than for areas farther north, it is believed that such an adjustment to the values of rainfall duration in Table 3 would reveal a closer relation between duration of surface run-off producing rain and period of rise.

Table 4 gives the apparent period of rise, P, for the distribution graphs of Figs. 3 and 4 as scaled from the diagrams. There may be some errors and mistakes in these values due to difficulty in distinguishing between the individual graphs. The duration of rainfall, as listed in Table 3, and the value of P adopted by the author for each of the two basins is also given. The data show some relation between duration of rainfall and period of rise and it is believed the relation would be more pronounced if periods of initial loss were deducted from the values of duration of rainfall.

TABLE 4.—COMPARISON OF RAINFALL DURATION AND PERIOD OF RISE

Description	Stream No. 9; Coweeta; P = 75 Min					Stream No. 1; Copper Basin; P = 38 Min			
	April 24, 1935	July 12, 1936	August 19, 1936	August 24, 1936	August 28, 1936	June 8, 1935	June 21, 1935	July 5, 1935	May 12, 1936
Duration of rainfall, in minutes	42	35	25	20	75	25	55	30	60
Period of rise (from Figs. 3 and 4), in minutes	103	53	66	77	110	39	58	36	46

The best evidence that the unit of time used with unit hydrographs has an appreciable effect on the results and that the unit should be considerably less

than the period of rise is given by the author under the heading "Applications of the Distribution Graph: Application of the Pluviagraph." It is stated that the interval or unit of time for the distribution graph is limited to such a length that the computations will give a smooth and correct curve. These same conditions undoubtedly controlled the break-down of the continuous precipitation into units of time. To accomplish this the author has used a unit of 30 min for Stream No. 2, Copper Basin; yet the period of rise is given as 60 min. The unit of time adopted for Stream No. 7, Coweeta Basin, is 30 min, and the period of rise is 60 min. The only other values given in the paper are for Stream No. 3, Bent Creek, for which the precipitation unit of time is 5 min, and the period of rise is 12 min.

The paper includes a very interesting discussion and illustration of variation in run-off coefficients. In considering the use of the pluviagraph to study the continuous variation in run-off coefficients (assuming the procedure is theoretically correct), it should not be overlooked that the factor obtained by dividing an observed rate of discharge by the corresponding pluviagraph rate is not the true run-off and rainfall ratio for any particular unit period of time except those for the first and last periods of a storm. The coefficient obtained by dividing the discharge rate by the pluviagraph rate is the weighted average of the actual ratios for the various unit periods, parts of the rainfall of which have been combined to obtain that particular pluviagraph rate. This fact does not necessarily impair the usefulness of the ratios between discharge and pluviagraph rates for comparative purposes, but it should not be disregarded when correlating the ratios with actual physical data.

The author has made an excellent analysis of what is undoubtedly a comprehensive collection of data. The availability of data such as those used in the analysis, and those being collected at numerous other experimental areas, suggests that it may not be long before the oft-heard statement decrying the lack of data must be replaced by one emphasizing the need of competent analysis.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

COMBINING GEODETIC SURVEY METHODS WITH CADASTRAL SURVEYS

Discussion

By Philip Kissam, Assoc. M. Am. Soc. C. E.

Philip Kissam,[9] Assoc. M. Am. Soc. C. E. (by letter).[9a]—The method of cadastral surveying described in this paper might well be used as a standard wherever cadastral surveys are made simultaneously over a large area. The paper is so inclusive and so well condensed that it might almost serve as a manual for this type of surveying.

There are a number of points that Mr. Berry describes which should be emphasized strongly and which are sometimes neglected in surveys of this kind. First, it is evident that all control survey points are monumented and marked with identifying numbers; descriptions of them are written and placed in a permanent record. This means, of course, that the survey is permanently located on the ground and it can be rerun for the purpose of locating any points that may be lost. Second, the work has been done according to the specifications of the U. S. Coast and Geodetic Survey, which means that the precision of the control is such that it can be made part of the fundamental triangulation net of the United States and, therefore, is available for use by any surveying agency. Third, the results have been computed in terms of the State plane co-ordinate systems, which means that there are available thousands of monumented positions within the area covered by the survey which have been determined in terms of these co-ordinates and therefore are useful as a basis for all types of surveying without the knowledge of geodetic reduction. Since the entire control system has been based on the accepted fundamental datum of the United States, the plane co-ordinates of these stations are the accepted plane co-ordinates recommended by the Federal Bureau of Surveys and Maps and contain an exceptionally low-scale correction.

With an arrangement of this kind the average land surveyor will be able to determine the State plane co-ordinates of all points on his surveys within

Note.—This paper by Carl M. Berry, Assoc. M. Am. Soc. C. E., was published in September, 1939, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: December, 1939, by William L. Sawyer, Assoc. M. Am. Soc. C. E.

[9] Associate Prof., Civ. Eng., Princeton Univ., Princeton, N. J.

[9a] Received by the Secretary December 1, 1939.

reasonable distance of the points thus established. Thus, all surveys in that area can be easily referred to the same datum. The control points cannot be lost, and the positions of property lines can be so described that they are permanent in position. The advantages of a definite standard plane rectangular datum are so numerous that the decision to use such a system cannot be overemphasized.

The adoption of methods that tend toward the reduction in costs as well as the improvements in accuracy are excellent, and the careful cost accounting method used should be emulated. Of particular interest is the method of using long tapes—that is, tapes from 200 to 300 ft long for third-order traverse. Utilization of long tapes is a great timesaver, and it is especially important that the taped distances are corrected for slope rather than an effort made to hold the entire tape in a horizontal position. The simplicity of Table 1 makes possible the field reduction of all measurements and eliminates the necessity for too rigid standardization of methods of support which are necessary when computation is made entirely in the office. It is believed that further economies could have been obtained had a more modern, simultaneous-coincidence, micrometer transit been substituted for the 10-sec repeating instrument used for second-order triangulation. Accuracy, speed, and reduction in the first costs would probably have been obtained, if such a transit had been used, together with the advantages of using an instrument which would have been far easier to transport. The utilization of plane co-ordinates for practically all computation subsequent to the determination of geodetic position for first-order and second-order triangulation and control traverse is to be commended, and attention is drawn to the reduction in computation costs which this provides without, however, departing from a standard datum.

The right-of-way plan illustrated in Fig. 9 is exceptionally complete. However, there is no note or legend which indicates whether the bearings given are true bearings, or grid bearings, and it would seem proper that grid bearings only should be used and so noted.

The entire survey seems to have been well thought out and the administrative details nicely standardized. Mr. Berry is to be complimented upon his fine contribution.

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DISCUSSIONS

GENERAL WEDGE THEORY OF EARTH PRESSURE

Discussion

By Howard F. Peckworth, M. Am. Soc. C. E.

Howard F. Peckworth,[15] M. Am. Soc. C. E. (by letter).[15a]—In the section of this paper devoted to "Pressure Exerted by Cohesive Earth," the writer quite agrees that there are many conditions under which Coulomb's equation does not give even approximate values. The writer would go still further than Professor Terzaghi and suggest that Equation (4) be abandoned entirely. The writer also agrees that, for cohesive soils, the prospects of predicting the lateral pressure successfully by theory are slight. In that case, should one not continue along the line of observations and design methods used and proposed by such men as the late Sir Benjamin Baker,[16] Hon. M. Am. Soc. C. E., and the late J. C. Meem,[7, 17] H. G. Moulton,[8] and E. G. Haines,[7, 8] Members, Am. Soc. C. E.?

The aforementioned engineers all discussed the special case in which the sheeting is driven from the ground downward and not constructed from the bottom upward with backfilling placed after the sheeting is completed. As in the present case, their observations applied to cohesive earth, with lateral pressure exerted against a sheeted trench, typical in the construction of sewers, foundations, and subways. Their observations can be summarized in three statements, as follows:

A. The line of rupture (see Fig. 10) and its relation to the depth of the excavation are independent of the angle of respose. Actually, there is no such thing as an angle of repose in this special case.

Note.—This paper by Karl Terzaghi, M. Am. Soc. C. E. was published in October, 1939, *Proceedings*. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion of the paper.

[15] Res. Engr. Insp., PWA, Santee-Cooper Project, Moncks Corner, S. C.

[15a] Received by the Secretary November 14, 1939.

[16] *Minutes of Proceedings*, Inst. C. E., Vol. LXV, p. 140.

[7] "The Bracing of Trenches and Tunnels, with Practical Formulas for Earth Pressures," by the late J. C. Meem, *Transactions*, Am. Soc. C. E., Vol. LX, June, 1908, p. 1.

[17] "Stresses in Cofferdams and Similar Structures," by the late J. C. Meem, *Civil Engineering*, December, 1934, p. 639.

[8] "Earth and Rock Pressure," by H. G. Moulton, *Transactions*, Am. Inst. of Mining and Metallurgical Engrs., 1920.

B. The line of rupture is independent of the degree of cohesion in the material.

C. The failure of material, in excavating operations ranging from moist sand to hard rock, appears to be a function of unit weight and characterized by an action independent of the nature of the material, provided the depth reached is sufficient to develop pressures in excess of its shearing strength.

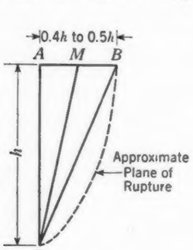

FIG. 10

Three examples may be cited to support the rather broad claim in statement *C*:

Example (1).—Sir Benjamin Baker stated, from observations on about 34 miles of deep timbered trenches and tunnels, that on each side "The slope of these fissures was so uniform at the angle of $\frac{1}{2}$: 1, measuring from the bottom of the excavation, that the Resident Engineer professed to be able to foretell with certainty where a building or fence wall, standing over the tunnel, would crack most."

Example (2).—There are five cases (observed by Mr. Moulton[8]) to support example (2): (*a*) On section 1*A* of route 12 on the double-track tunnel under Flatbush Avenue, Brooklyn, N. Y., a decided break occurred in the ground on either side of the tunnel, at a distance out from the net line, of one half the distance from the surface to the top of the tunnel, at the net line; (*b*) gravel banks in placer gold mining operations in Klamath County, Oregon, and Trinity and Shasta counties, California, stood on a 1 vertical : 0.5 horizontal slope; (*c*) placer gold mining operations near Dawson, Yukon, Canada, where the gravel was cemented by ice, also stood on a 1 vertical : 0.5 horizontal slope; (*d*) in stoping operations at Ray, Ariz., Globe, Ariz., and Ely, Nev., the break in the ground occurred on a 1 vertical : 0.5 horizontal slope out from the edge of the stoping; and (*e*) even on such a material as sawdust (in a sawmill at Klamath Falls, Ore., October, 1919) when removed from the bottom of the pile by bulldozers, the sawdust broke at the top on a slope of 1 vertical : 0.5 horizontal. Case (*d*) showed that hard stratified rock, overlain with earth, followed the same law as the gravel.

Example (3).—In 1908, Mr. Meem[7, 17] proposed a preliminary wedge theory based on the assumptions built around the angle of repose. At that time he realized that practice did not require rangers to be as heavy as his theory demanded; and by 1934 he had revised his preliminary wedge theory, discarding the angle of repose and building it around the intersection of the plane of rupture and the ground which Messrs. Baker, Moulton, and Haines had placed at one half the depth (provided the depth was great enough to develop pressures in excess of the shearing strength of the material) and which Mr. Meem placed at 40% of the depth.

The practical results of observations *A*, *B*, and *C* were expressed by Mr. Meem as follows: "All the soil in the area *A B C* [Fig. 10] is undoubtedly supported by the face *A C* on one side, and the mean plane of repose [rupture] *B C*,

on the other. If it be arbitrarily assumed that one half of this wedge causes pressure against the face *A C*, and that the other half rests on *B C*, then the three problems that must be determined are: 1 * * * the mean plane of repose [rupture] *B C*, or the angle *A C B*; 2 * * * the rational position of the mid-plane *M C*; and 3 * * * the elements and measure of thrust due to the area or volume, *A M C*." Whereas Messrs. Baker, Moulton, and Haines had set the ratio *A B* : *A C* as 0.50 : 1, Mr. Meem set it at 0.4 : 1.

Between 1926 and 1934 the writer had occasion to design timbering and bracing using the foregoing method. Observations on this timbering were that, when a crane (surcharge) was placed next to the cut and was used to set steel in the excavation below, timbering designed on the 1-on-0.4 theory showed distress, but timbering designed on the 1-on-0.5 theory did not. This indicated to the writer that the theories of Messrs. Meem, Baker, Moulton, and Haines were not far from correct.

To illustrate how carefully the foregoing condition must be distinguished from the case of a retaining wall with backfill behind it, a case may be cited in which a sheeted subway cut was intersected by a large runway to provide access for trucks from the bottom of the excavation to the street. It became necessary to close this access, and sheeting and bracing were placed across the opening, in all respects similar to the sheeting and bracing (which was entirely adequate) on each side of the access runway. Cohesive backfill was then dumped behind this sheeting. In the next few weeks, the sheeting, and the bracing opposite the backfill, moved inward 9 in. and had to be reinforced heavily to prevent entire failure.

Another example is that of a subway in which a spur line branched off from the main line in a wide curve. The main line was built first and the load of the sides transferred to the finished subway structure before the branch line was started. The same sheeting and bracing were used on the branch as on the main line and it was constructed toward the main line. When the line of rupture of the branch line intersected the line of rupture of the main line, the bracing showed signs of distress and much heavier timbering had to be substituted in order to bring the branch alongside and into the main line.

The writer would like to emphasize the author's suggestion that the Soil Mechanics and Foundations Division collect data on actual cuts such as those described by Professor Terzaghi and as proposed by Mr. Moulton in 1920[18] and by the writer in 1932.[19]

To recapitulate the writer's views—for cohesive earth, in the special case of sheeting and bracing driven from the ground downward (not the case of a retaining wall with backfill placed behind it):

(1) The author is correct in stating that Equation (4) yields only very approximate values;

(2) The author is also correct in stating that the prospects of being able to predict the lateral pressure successfully by theory are slight; and

[18] "Earth and Rock Pressure," by H. G. Moulton, *Transactions*, Am. Inst. of Mining and Metallurgical Engrs., 1920, p. 20.

[19] *Civil Engineering*, September, 1932, p. 585.

(3) The writer proposes that this paper would not be complete without discussion and experiments proving or disproving observations *A*, *B*, and *C*, as described in this discussion.

The writer has confined his discussion to cohesive earth because in actual practice he has dealt with it more often than with absolutely cohesionless material. Acceptable definitions for the following terms would help clarify this matter: Active earth pressure, passive earth pressure, angle of repose, angle made by material pushed off the slope from the top, and the angle made by the line of rupture and the face of a sheeted trench.

In conclusion the writer wishes to thank Professor Terzaghi for his excellent paper.

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DISCUSSIONS

AN IMPROVED METHOD FOR ADJUSTING LEVEL AND TRAVERSE SURVEYS

Discussion

By Messrs. George H. Dell, and Howard S. Rappleye

George H. Dell,[5] Assoc. M. Am. Soc. C. E. (by letter).[5a]—A clear and interesting description of a simplified adaptation of the junction point method to the adjustment of level and traverse nets is presented in this paper. It may be worth noting that the results given by this method are identical with those obtained by basing the adjustment upon appropriate circuit-closure and route-closure conditions. It should also be observed that in applying the junction point method it is necessary that at least one point in the net be treated as a fixed point, or otherwise an indeterminate system of equations will result.

The Doolittle method of solving the normal equations is unexcelled for use in offices where the execution of intricate adjustments is a major occupation. In fact, axisymmetrical systems of equations are so frequently encountered in various branches of engineering that it would be highly desirable to include the Doolittle method in the mathematics requirements of all engineering students.

For organizations or individuals having to deal only occasionally with adjustments of the type under discussion, or for classroom work where an adequate supply of computing machines may not be available, the solution of the equations by successive convergence, with slide rule, offers a satisfactory substitute for the more precise Doolittle method. Such a solution is illustrated in Fig. 5, which deals with the latitude corrections in the example given by the authors. The absolute term at each junction is shown within a square, followed by the sum of the weights, ΣP. In the first cycle, the absolute terms, augmented by corrections carried over from adjacent junction points, are distributed toward these junction points in proportion to the weights of the con-

Note.—This paper by Clarence Norris, Esq., and Julius L. Speert, Assoc. M. Am. Soc. C. E., was published in October, 1939, *Proceedings*. This discussion in printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion of the paper.

[5] Associate in Civ. Eng., Univ. of Illinois, Urbana, Ill.

[5a] Received by the Secretary November 20, 1939.

necting lines. In succeeding cycles, only the preceding set of corrections are brought over and distributed. In each cycle the sequence of adjustment follows that of the authors' equations.

Starting with junction 1, $\frac{1.042}{3.51}$ of 5.59 (= 1.66) is distributed toward junction 2. At junction 2, the quantity to be distributed is $1.66 - 4.86 = -3.20$, the distribution toward junctions 1, 3, and 5 being -1.22, -1.01, and -0.97,

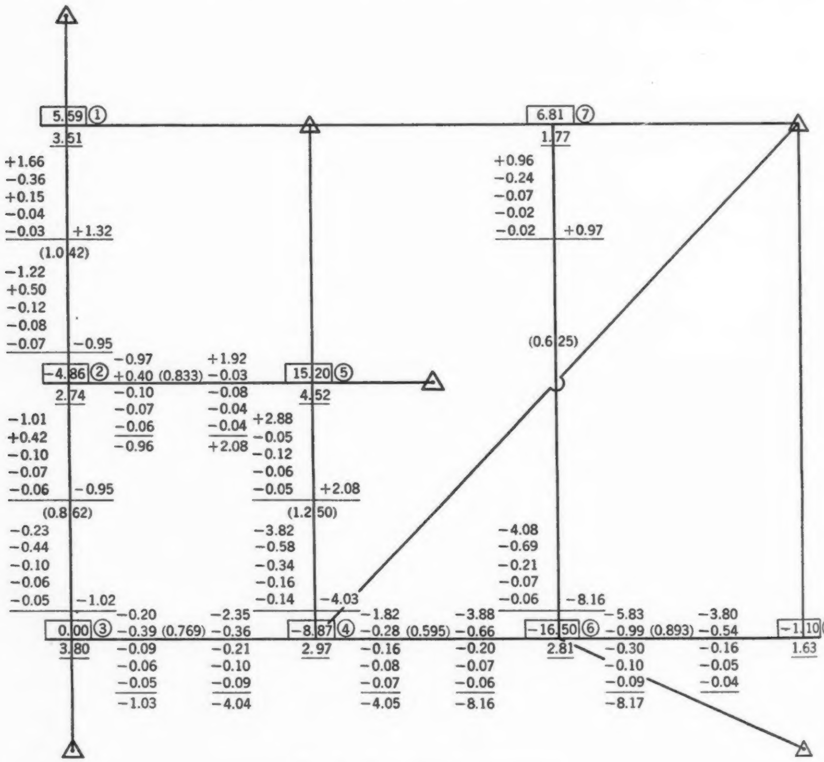

FIG. 5.—LATITUDE ADJUSTMENTS

respectively. Similarly, at junction 3, the quantity to be distributed is $-1.01 + 0.00$, or -1.01, of which $\frac{0.862}{3.80}$ ($= -0.23$) and $\frac{0.769}{3.80}$ ($= -0.20$) are distributed toward junctions 2 and 4, respectively. A complete list of the quantities to be distributed is given in Table 6, and it is seen that after $4\frac{1}{8}$ cycles all equations have begun to converge consistently. The fifth set of corrections in Fig. 5 is the result of series summations, obtained by multiplying the fourth set of corrections by the summation factor, which is found as follows: The absolute sum of the last set of eight quantities in Table 6 (= 1.67) was divided by the

absolute sum of the preceding eight quantities (= 3.59), giving 0.465 as the average rate of convergence, r. The summation factor, $\frac{r}{1-r}$, therefore, is 0.87. The final values shown in Fig. 5 are the unknowns of the adjustment equations and are obtained by dividing the sums of the various columns by the weights of the corresponding lines.

With regard to the opening paragraph of the authors' "Conclusion," the writer agrees that mathematical inconsistencies should be eliminated by adjustment, and that careful surveys are thereby improved. The writer, however, is inclined to guard against placing undue reliance on such improvement, and believes that a high standard of precision can best be insured by rigorous field work. The results of a least-squares adjustment depend largely upon the weights, and, in a quantitative sense, surveys of high order are much less seriously affected by errors in judging the weights than are surveys of relatively low order.

TABLE 6.—LIST OF QUANTITIES DISTRIBUTED

Cycle	Junctions in Fig. 5*							
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
1	+5.59	−3.20	−1.01	−9.07	+10.41	−18.32	+2.73	−6.93
2	−1.22	+1.33	−1.93	−1.39	−0.18	−3.12	−0.69	−0.99
3	+0.50	−0.32	−0.46	−0.80	−0.44	−0.94	−0.21	−0.30
4	−0.12	−0.22	−0.28	−0.38	−0.23	−0.31	−0.07	−0.10
5	−0.08	....	....	....	....	....	....	....

* In Fig. 5 the junction numbers are encircled.

The traverse used by the authors has been analyzed by the writer with a view to determining the effects of errors or variations in the weights. As a basis for this study, use was made of expressions for the differentials of the unknowns of the normal equations with respect to the weights,[6] the calculations being based on hypothetical variations of 10% in the weights. The individual effects of these variations in weight are shown in Table 7, quantities of about 0.05 ft or greater being underlined, and the maximum combined effects being shown in the final line. For example, a 10% increase in the weights of lines d and e would produce corresponding changes of + 0.2366 ft and − 0.1590 ft, respectively, in the value of C_{1N}, and the maximum combined effect of such errors in all weights would be 0.5036 ft for this particular unknown.

Because of the complexity of the factors involved, it appears not unreasonable to suppose that, unless great care is taken, the inaccuracies in the assumed weights of some of the lines may be as great at 50%. The effects of variations greater than 10% in the weights would be approximately proportional to the quantities shown in Table 7. Therefore, it is seen that variations of 50% in the weights of certain individual lines would produce corresponding variations of from 0.25 ft to 1.00 ft or more instead of the underlined quantities, and the maximum combined effects of such errors or variations would range from 0.94

[6] *Bulletin No. 309*, Sections 7, 10, and 13, Univ. of Illinois Eng. Experiment Station, Urbana, Ill.

TABLE 7.—DIFFERENTIAL CHANGES (IN HUNDREDTHS OF A
INCREASE IN

Line	dC_{1N}	dC_{1E}	dC_{2N}	dC_{2E}	dC_{3N}	dC_{3E}	dC_{4N}
a	+0.86	−0.11	+2.93	−0.38	+7.06	−0.91	+2.49
b	−0.05	−1.56	−0.18	−5.26	+0.09	+2.64	+0.01
c	−4.28	−2.51	+8.29	+4.87	+2.18	+1.28	+1.46
d	+23.66	+13.80	+10.62	+6.22	+2.78	+1.62	+1.87
e	−15.90	−9.25	−7.14	−4.17	−1.87	−1.09	−1.26
f	−0.04	−0.04	−0.12	−0.12	−0.16	−0.16	−0.63
g	−0.29	−0.14	−0.98	−0.49	−4.68	−2.34	+7.78
h	−0.19	−0.68	−0.64	−2.28	−0.81	−2.89	−3.29
i	−0.03	−0.10	−0.09	−0.36	−0.12	−0.46	−0.47
j	−0.46	+0.23	−1.55	+0.77	−1.96	+0.98	−7.93
k	−0.02	−0.09	−0.08	−0.29	−0.10	−0.37	−0.39
m	−0.02	+0.08	−0.06	+0.27	−0.08	+0.34	−0.33
p	+0.02	+0.16	+0.08	+0.54	+0.11	+0.69	+0.43
q	−0.10	+0.08	−0.34	+0.27	−0.43	+0.34	−1.75
r	−1.11	−0.76	−3.75	−2.54	−1.77	−1.20	−4.55
s	−0.08	−0.02	−0.25	−0.06	+0.50	+0.12	+2.78
t	−2.63	+0.05	−8.88	+0.16	−1.77	+0.03	+1.23
u	+0.62	+0.84	+2.09	+2.85	+0.99	+1.34	+2.54
Maximum	50.36	30.50	48.07	31.90	27.46	18.80	41.19

ft to 2.67 ft, approximately. However, if it were possible to limit the errors in the weights to the order of 10%, the corresponding errors in the final coordinates could not be materially in excess of 0.50 ft, and would probably be within 0.25 ft in most of the quantities.

It is thus seen that, in order to use the least-squares adjustment to best advantage as a means of improving the results of a low-order survey at a saving in field costs, it is desirable that great care should be exercised in determining the weights. Among the factors upon which the weight of a given line depends are: (*a*) The length of line, (*b*) the number of instrument stations or set-ups it contains, (*c*) the number of independent measurements (repetitions) entering into the data, (*d*) atmospheric conditions, (*e*) the character of the terrain, and (*f*) the quality of instruments and survey methods, including the personal equations or relative skilfulness of the various observers. It would appear that the development of a practical method of taking these factors into account in determining the weights would be a valuable step.

The authors are to be highly commended for presenting such a clear description of their improved junction point method and for their success in producing a really useful simplification of the method in question.

Corrections for *Transactions:* In Tables 3 and 4, at the bottom of Columns N and E, the signs of each of the last three items should be reversed; on page 1374, lines 34, 36, and 41, the signs of the numbers 3.741, 2.992, and 0.749 should be reversed in each case; on page 1376, lines 18, 19, 24, and 26, change 0.01, 0.01, 0.00, and 0.007, to 0.02, 0.02, − 0.01, and 0.009, respectively; and on page 1381, in Equation (12) change P_{ad}, which appears twice, to P_{ab}.

FOOT) IN THE UNKNOWNS C_{1N}, C_{1E} $\cdots$ C_{8N}, C_{8E} DUE TO 10% WEIGHTS OF LINES

dC_{4E}	dC_{5N}	dC_{5E}	dC_{6N}	dC_{6E}	dC_{7N}	dC_{7E}	dC_{8N}	dC_{8E}
−0.32	+1.23	−0.16	+0.70	−0.09	+0.25	−0.03	+0.38	−0.05
+0.33	−0.03	−0.88	+0.00	+0.09	+0.00	+0.03	+0.00	+0.05
+0.86	+1.93	+1.13	+0.41	+0.24	+0.14	+0.09	+0.23	+0.13
+1.09	+2.48	+1.45	+0.53	+0.31	+0.19	+0.11	+0.29	+0.17
−0.73	−1.67	−0.97	−0.36	−0.21	−0.13	−0.07	−0.19	−0.11
−0.63	−0.20	−0.20	−2.54	−2.54	−8.81	−8.81	−1.40	−1.40
+3.89	+1.97	+0.98	+2.21	+1.10	+0.78	+0.39	+1.21	+0.60
−11.72	−1.03	−3.66	−0.93	−3.33	−0.33	−1.17	−0.51	−1.82
−1.84	−0.15	−0.57	−1.90	−7.41	−0.67	−2.62	−5.19	−20.23
+3.98	−2.48	+1.24	+9.51	−4.76	+3.36	−1.69	+5.21	−2.62
−1.52	−0.12	−0.47	−1.56	−6.08	−0.55	−2.16	+3.28	+12.79
+1.39	−0.10	+0.43	−1.30	+5.59	−4.54	+19.39	−0.72	+3.07
+2.79	+0.13	+0.87	+1.73	+11.20	+0.61	+3.96	+0.95	+6.14
+1.39	−0.55	+0.43	−7.05	+5.58	+9.52	−7.54	−3.86	+3.06
−3.09	−9.10	−6.18	−1.29	−0.88	−0.46	−0.31	−0.71	−0.48
+0.68	−1.26	−0.31	+0.79	+0.19	+0.28	+0.07	+0.43	+0.11
−0.02	+3.85	−0.07	+0.35	−0.01	+0.13	−0.00	+0.19	−0.00
+3.46	+5.08	+6.91	+0.72	+0.98	+0.25	+0.35	+0.39	+0.54
39.73	33.36	26.91	33.88	50.59	31.00	48.79	25.14	53.37

HOWARD S. RAPPLEYE,[7] M. AM. SOC. C. E. (by letter).[7a]—After all is said and done, what appears simplest to each individual is that with which he is most familiar. The method of adjustment proposed in the paper by Messrs. Norris and Speert undoubtedly appears to be simple to them because they are thoroughly familiar with it; yet the writer, who attempted to duplicate a previously completed adjustment of a simple level net by the new method, as a means of comparison, was not at all fully convinced that the statement (see "Conclusion")—"* * * if the instructions are observed carefully, a complete adjustment can be made by a clerk familiar with the use of the computing machine"—was not slightly overdrawn. Now that he has actually made an adjustment by the method, and has found that the results agreed within one unit in the last place with the results of the previous adjustment by another method, the writer can see that the proposed method has much to recommend it.

Henry G. Avers, Assoc. M. Am. Soc. C. E., has presented[8] the complete adjustments of a level net, including six closed circuits and either one or two fixed elevations, solved by both the condition-equation and observation-equation methods. As a check on the method proposed by Messrs. Norris and Speert, this same net was adjusted by their method, and it was found that the resulting table of adjustment equations, except the "constant-term" and "check-sum" columns, was identical with that used in the observation-equation method[9] with all signs changed.

[7] Chf., Section of Leveling, Div. of Geodesy, U. S. Coast and Geodetic Survey, Washington, D. C.

[7a] Received by the Secretary December 7, 1939.

[8] "Manual of First-Order Leveling," by Henry G. Avers, *Special Publication No. 140*, U. S. Coast and Geodetic Survey, pp. 53–74.

[9] *Loc. cit.*, p. 73.

As to the relative speed of the various methods of adjustment, the writer is not yet sufficiently familiar with the Norris-Speert method to make an adequate comparison; but at present he fails to see any marked advantage of the proposed method over the older methods in this respect.

The fact that in the proposed method practically the same routine is used for the adjustment of both horizontal and vertical control surveys is an advantage. The routine is quite mechanical and, once one becomes familiar with the details of the procedure, little or no thought need be given to the underlying mathematics. A strict attention to algebraic signs, and care in the manipulation of the computing machine, should lead to excellent results.

However, the writer would like to caution any one who is attempting for the first time to adjust a traverse or level net by this method, and with no help other than the paper as printed, to select a small adjustment for practice purposes. Otherwise, failure to follow all the exacting detail may lead to erroneous results and eventual abandonment of a perfectly good method as being "too complicated."

In the writer's opinion a sample of a small level-net adjustment should have been included in the paper. He suggests that it be included in the closing discussion. It should be carried at least as far as the formation of the complete set of normal equations. An explanation of the application of the resulting corrections in the case of a level-net adjustment should also be included in order to make the paper more generally useful to those who may want to adjust levels as well as traverse.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

RELATION OF THE STATISTICAL THEORY OF TURBULENCE TO HYDRAULICS

Discussion

BY HUNTER ROUSE, ASSOC. M. AM. SOC. C. E.

HUNTER ROUSE,[19] ASSOC. M. AM. SOC. C. E. (by letter).[19a]—In welcoming the appearance of this latest interpretation of present-day theories of fluid turbulence, one cannot help being impressed by the rapidity with which the modern engineer profits by advances made in related fields of science. It was many decades before hydraulic engineers became fully aware of the value to them of the fundamental contributions of Boussinesq and Reynolds, whereas engineers of today already have before them in a clear, concise form an interpretive summary of the many significant developments made within the last few years. It is particularly fitting that this paper has been written by Professor Kalinske, for he is the first of the hydraulic engineers to have applied these statistical theories in the laboratory to problems primarily hydraulic in nature.

As is evident from even casual study, the readers for whom the paper is particularly intended are those whose efforts may immediately benefit by the adoption of the statistical method of approach—that is, research men seeking to further the science of hydraulics. (As if in illustration of the value of these methods to the research profession, the author, following the preparation of the present paper, collaborated in the application of the statistical approach to the analysis of the amount of material lifted from the bed of a sediment-bearing stream[20]—with results likely to prove of great worth in the ultimate solution of the suspended-load problem.) However, the writer feels that the practicing engineer may also find much of interest in this paper, particularly if he overcomes a natural tendency to let its mathematical aspects conceal the general information which it contains.

In past résumés of turbulence theories, it will be recalled that the subject matter was necessarily restricted to conditions of statistically steady, uniform

NOTE.—This paper by A. A. Kalinske, Esq., was published in October, 1939, *Proceedings*. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion of the paper.

[19] Prof. of Hydraulics, Univ. of Iowa; Cons. Engr., Iowa Inst. of Hydr. Research, Univ. of Iowa, Iowa City, Iowa.

[19a] Received by the Secretary November 25, 1939.

[20] "The Relation of Suspended to Bed Material in Rivers," by E. W. Lane, M. Am. Soc. C. E., and A. A. Kalinske, *Transactions*, Am. Geophysical Union, Part IV, 1939, p. 637.

flow, such as that in circular pipes, the growth of turbulence in an expanding boundary layer receiving only brief attention. The internal mechanism of the turbulence was investigated qualitatively, but only as a means of developing general expressions for velocity distribution and boundary resistance. Although the resulting equations were sufficiently exact for practical use, the qualitative analysis of the turbulence on which they were based was not accurate enough to establish the absolute intensity of the turbulence at any arbitrary point in the flow. That knowledge of this nature has become essential to further progress is shown by the attention now being given to quantitative analysis of turbulence in such widely different fields as meteorology, oceanography, and aeronautics—to which fields that of hydraulics is now added.

For the hydraulic engineer it is no longer sufficient merely to acknowledge the presence of turbulence in uniform circular pipes. He must realize, as Professor Kalinske points out, that such terms as "shock loss" and "impact loss" are definitely misnomers, that all hydraulic losses are ultimately attributable only to viscous dissipation into heat, and that by understanding the formation and subsequent decay of turbulence such losses can be controlled to a great extent. Such an understanding is probably instinctive to many an hydraulic engineer, although on every side one encounters the results of basic misconceptions. For instance, it has been suggested that one must devise a new word to distinguish the large-scale "turbulence" in rivers from the small-scale turbulence in pipes—although the large-scale "turbulence" in question really consisted of local eddies bearing the same relation to the actual turbulence of the river as the eddies at a change in pipe section bear to the normal turbulence of pipe flow. As noted by the author, eddies produced by local disturbances are by no means synonymous with true turbulence. Such eddies, however, are a primary source of turbulence, and the rate of subsequent energy dissipation is a matter of considerable importance. For example, many so-called "energy dissipators" could be made far more effective and less dangerous in their downstream influence if designers realized not only that the formation of large-scale eddies effectively reduces the mean velocity of flow, but also that the presence of such eddies is sometimes as harmful as that of a high mean velocity; as pointed out by Professor Kalinske, the production of intense small-scale eddies (approaching the state of isotropic turbulence) is an essential means of hastening energy dissipation.

Of primary significance, evidently, is the fact that both the length scale and the velocity scale of turbulence govern the processes of diffusion and energy dissipation. The product of these scales is already familiar to the student of turbulence in the factor ϵ, the so-called "eddy conductivity," previously determinable only for uniform flow, and then only roughly through measurement of velocity distribution and resistance. Professor Kalinske has outlined two methods whereby the length and velocity scales for both uniform and non-uniform flow may be determined individually. He notes, however, that in either method the evaluation of the results is extremely tedious. Several hydraulic laboratories are already on the search for a simpler, and hence more practical, means of accomplishing such measurement in water, but their goal is by no means yet in sight. It is to be hoped that the present paper will also serve as a further incentive to this end.

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DISCUSSIONS

EFFECTIVE MOMENT OF INERTIA OF A RIVETED PLATE GIRDER

Discussion

BY MESSRS. CHARLES M. SPOFFORD, E. MIRABELLI, EDWARD GODFREY, WALTER H. WEISKOPF, AND C. D. WILLIAMS

CHARLES M. SPOFFORD,[15] M. AM. SOC. C. E. (by letter).[15a]—The writer has read this paper with much interest and wishes to commend the authors for their careful investigations and excellent report. That such investigations are needed is shown clearly by the fact that standard specifications for plate-girder design in the United States are not yet stabilized with respect to the allowance for rivet holes or the position of the center of gravity.

The authors' experimental method of determining the position of the center of gravity of the girder was such as to give the average moment of inertia over the central part of the girder only. In this part of the girder the bending moment is uniform, no web splice occurs, and there are no vertical lines of rivets except at the ends of the section. The results show clearly that under these conditions the centers of gravity of the girders tested lie nearer midheight than when determined by computations of the net section based upon deducting rivet holes in the tension part of the girder only. The calculated values given by the authors, however, give only the location of the center of gravity of a section through rivets, whereas the average center of gravity would be higher than this since rivet holes do not occur at every section.

For the girders tested, with rivets at 5-in. pitch in the vertical legs of the flange angles only, the position of the average center of gravity would be considerably greater than that computed by the authors and would conform quite closely to observed values. This would apply in lesser degree in cases where the rivet pitch is reduced to 2.5 in.

The writer feels that the results of the tests are scarcely sufficient to warrant definite conclusions for use in actual design. The girders tested were somewhat

NOTE.—This paper by Scott B. Lilly, M. Am. Soc. C. E., and Samuel T. Carpenter, Assoc. M. Am. Soc. C. E., was published in October, 1939, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: December, 1939, by Messrs. William R. Osgood, Clyde T. Morris, B. R. Leffler, E. Neil W. Lane, Lewis E. Moore, W. E. Black, and L. E. Grinter.

[15] Hayward Prof. of Civ. Eng., Mass. Inst. Tech.; Cons. Engr. (Fay, Spofford and Thorndike), Boston Mass.

[15a] Received by the Secretary November 14, 1939.

out of line with those generally employed in practice, since the section modulus of the web was much larger in proportion to the total value of the moment of inertia than is common in actual plate girders, particularly in bridge girders. This condition is especially true in the part of the girder in which the measurements were made, since the shear in this part of the girder was zero. Moreover, the weakest section of a large plate girder is generally at a web splice, and it would have been desirable if the authors could have included some tests for the value of *I* and the position of the center of gravity of the girder at such a splice.

The authors' conclusion that the data from these tests point to the acceptability of using the gross moment of inertia in design involves the conclusion that either the allowable unit stress should be reduced, or that rivet holes do not weaken a girder, which latter conclusion the writer believes designers will be slow to accept. It is to be hoped that Messrs. Lilly and Carpenter will find it possible to make additional tests along these lines.

E. Mirabelli,[16] M. Am. Soc. C. E. (by letter).[16a]—These tests show the effect of flange holes, bolts, and rivets on the deflection and on the average flange stress of a plate girder when subjected to pure bending by a static loading which induces average stresses within the elastic limit of the material. It is questionable if they show how the average flange stress at a critical section is affected by the presence of holes or rivets, particularly in a region subjected to the combined action of bending moment and shear. The tests are of definite value in contributing toward an understanding of plate-girder action, and there is no intention of disputing their importance. However, it is doubtful if this set of tests, by itself, provides enough information to form a sufficient basis for changing the current design practice of using net girder section. Following are some reasons why it appears to the writer that caution is warranted in abandoning the usual design procedure:

(1) The effective or observed moments of inertia given in Table 3 are based on observed deflections which are affected by all sections of the girder—those between rivet holes as well as those at the rivet holes. Such values of the effective moment of inertia are averages for the girder as a whole. They are proper ones to use in computing deflections, but are not correct for evaluating the stress at a critical section through rivet holes. The flange stress computed by the beam formula, using such a moment of inertia, is not the critical stress; it is some kind of an average stress. Similarly, the observed flange stresses that were measured with a 10-in. gage are the averages of stresses on all sections within the length of the gage. They are no indication of the average stress on a section passing through the rivet holes. It seems reasonable to assume that failure follows when the stress exceeds some critical amount on a single section without necessarily extending over an entire region. To prove this it is necessary only to test a perforated or riveted bar in tension and find that failure occurs on a section through the hole.

(2) The girders were tested in bending only, without simultaneous shear. Consequently, the flange rivets were not subjected to any stress due to flange-

[16] Associate Prof., Civ. Eng., Mass. Inst. of Tech., Cambridge, Mass.

[16a] Received by the Secretary November 17, 1939.

stress transfer which accompanies an increment in bending moment. Such rivets act simply as stitch rivets to hold the angles and cover plates together. Any stress that exists in the rivets is that due to initial tension in the shank caused by cooling, and whatever stress occurs in the rivet head through friction at the surface of contact with the plates or angles. The condition of the rivet is similar to that indicated in Fig. 8 in which a rivet is shown holding together four plates which are subjected to end forces, F. The only possible stress path across the gap, $G\ G$, is through the rivet heads. The strength of such a connection may be calculated if assumptions are made regarding the coefficient of friction under the rivet heads and the initial tension in the shank. The splicing action of the rivet heads, in some degree, makes up for the loss of section due to rivet holes in a plate girder and, to some extent, explains the increase in effective moment of inertia of the riveted specimen when compared with the specimen with open holes.

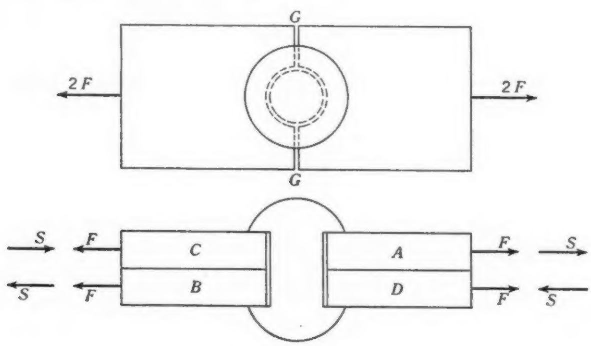

FIG. 8

Now, if the condition in Fig. 8 is changed by the introduction of shearing forces, S, the total tension in plates A and B will be $T_1 = F + S$, and in plates C and D will be $T_2 = F - S$. If $T_2 = 0$, there will be no stress to transfer from D to B or from C to A, although the total stress to be spliced is still $2\ F$. It is probable that some stress will pass from B to A by following a path which involves eccentricities; but it seems that the capacity of the rivet heads as a splice for direct stress is definitely reduced by the presence of the shearing forces. Now eliminate the gap, $G\ G$, by making C continuous with A, and B continuous with D, and the condition is similar to that occurring in the flanges of a girder. It seems reasonable to conclude that under this condition, also, the presence of shear will influence the amount of direct stress passing across the hole through the rivet head and thereby will weaken the section. In fact it has been found by test[17] that a stressed rivet has a greater effect in weakening a plate than has an open hole.

Extension of the authors' conclusions to girder sections stressed simultaneously in bending and shear would not seem justified. The presence of stressed

[17] "Tension Tests of Large Riveted Joints," by Raymond E. Davis and Glenn B. Woodruff, Members, Am. Soc. C. E., and Harmer E. Davis, Assoc. M. Am. Soc. C. E.; discussion by W. M. Wilson, M. Am. Soc. C. E., *Proceedings*, Am. Soc. C. E., October, 1939, pp. 1454 and 1462.

rivets in the flanges will reduce the effective moment of inertia. It is true that maximum bending moment and maximum shear do not occur at the same section except in cantilevers. However, maximum flange stress and maximum rivet shear may occur simultaneously at cross sections at the end of cover plates.

(3) The tests were made under static loading whereas, in service, girders are subjected to varying or pulsating loadings. The question of type of loading would not be involved if the rivet holes with the attending stress concentrations were not present. The effect of rivet holes differs with the two types of loading. This difference is shown by tests[18] which indicate that the tensile strength of a perforated or riveted bar under static loading is greater than that of a solid bar of the same net cross section (57,600, 59,700, and 62,300 lb per sq in. for solid, perforated, and riveted bars, respectively). However, the fatigue strength of the perforated and riveted bars is considerably less than that for the solid bar under a loading that pulsates from 2,820 lb per sq in. to the breaking stress (39,100, 29,900, and 24,300 lb per sq in. for solid, perforated, and riveted bars, respectively). In other words, the influence of stress concentrations may be disregarded under static loading but it cannot be disregarded under a vibrating loading. It is true that in a properly designed new structure the primary stress will not attain a value of 24,300 lb per sq in. and the condition is not nearly so bad as this value might indicate.

The stress concentrations adjacent to the rivet holes nullify the assumption of linear stress variation used in the ordinary beam theory and thereby lead to an observed moment of inertia which, under static loading, is larger than that computed for the net section. The difference between observed and computed moments of inertia for the "B" test sections (Table 3, Columns (1) and (4)) may be explained in part by this action. Under the action of a pulsating loading the aid given by the stress concentrations is diminished or lost entirely.

(4) These tests were not intended to reveal the useful carrying capacity of the girders—that is, the yield point of the girder as a whole, or the point at which the girder deflection increases rapidly with further application of loading. This is probably the proper point to which the margin of safety should be measured. If the girder deflection is in direct proportion to the loading until the useful carrying capacity is reached, the flange-stress intensity might be used as a measure of the factor of safety. Otherwise the factor of safety must be based on applied loading.

Due to the presence of stress concentrations at the rivet holes, it is quite possible that the linear relation between girder deflection and applied loading may cease to exist shortly after the stress in the vicinity of the rivet holes exceeds the yield point. This may occur while the average flange stress is still considerably within the elastic range. On the other hand, it is also possible that this linear relation may extend to a loading exceeding that at which the average flange stress is equal to the yield point stress. It would be interesting to know the relation between loading, strain gage reading, and deflection for the specimen PT-1 which was loaded to produce a flange stress of 30 kips per sq in.

[18] "Gemeinschaftsversuche zur Bestimmung der Schwellzugfestigkeit voller, gelochter und genieteter Stäbe," by K. Klöppel, *Der Stahlbau*, Berlin, June 19, 1936, pp. 97–112.

It may be quite desirable and safe to design on the basis of gross moment of inertia as recommended by the authors. The writer feels that such a procedure would evaluate actual conditions correctly for only a small percentage of cases, and that in most cases the procedure would provide an actual margin of safety which would be less than the assumed amount. However, considering the fact that secondary stresses in plate girders are likely to be small, it may be permissible to reduce the margin of safety. From this point of view the weakening effect of rivet holes would be treated as a secondary effect which would be covered by the margin of safety. Until the influence of holes and rivets in girders is established more definitely, it is the writer's belief that the customary practice of using net section should not be abandoned.

EDWARD GODFREY,[19] M. AM. SOC. C. E. (by letter).[19a]—Conclusion 3 is correct. It agrees with what the writer has long held to be proper in design—namely, that the neutral axis of a girder should be considered in the center of gravity of the gross cross section. Furthermore, the sentence in Conclusion 5, "design methods should be made as simple as possible," expresses what the writer has stated many times.

The last sentence in Conclusion 1 is not clear. It reads: "The net moment of inertia, used in design with top and bottom flanges alike, should yield conservative stresses." This does not seem to comport with the trend of the paper nor the other conclusions. The authors do not seem to be advocating the use of net moment of inertia in design but rather the reverse. The trend of the paper seems to be toward justification of the practice of using the gross moment of inertia. There is no question about the safety of design that uses the net moment of inertia.

Deflection measurements and extensometer measurements do not tell anything of value about the actual, or critical, stresses of perforated flanges of a girder or of perforated plates or angles in tension. These measurements show only the average stress in the length of piece where measurement is made. The extension beside the rivet hole is only a small fraction of that in the part not perforated, although the actual unit stress beside the hole may be very much greater than in the part not perforated.

Suppose, in a girder or I-beam, there were transverse saw slits instead of rivet holes. Deflection and extensometer measurements would be practically identical, in such members, with those of solid members of the same dimensions for stresses within the elastic limit. This is because the actual stretch beside the slit is only a very small fraction of the stretch between slits in the longitudinal direction. No one will contend that the members with the saw slits will be as safe as the solid ones.

The only difference between a plate perforated with saw slits and one perforated with rivet holes, so far as safety in tension is concerned, is that the former will probably break at, or a little above, the yield point, whereas the latter may hold until the usual ultimate stress is reached. The writer has made tests where ordinary steel specimens broke just above the usual yield point of

[19] Structural Engr., Pittsburgh, Pa.

[19a] Received by the Secretary November 24, 1939.

the net area beside punched holes, whereas pieces with drilled or reamed holes broke at the usual ultimate stress.

The writer is quite out of agreement with Conclusion 5. It is a damaging admission to say that stresses as ordinarily specified for structural work result in unit stresses "reaching" the yield point of the steel as the unit stress in the gross section "approaches" that specified. This can mean nothing else but that, when the allowed unit stress on the gross section is reached, the actual unit stress beside rivet holes exceeds the yield point. The argument has been that rivets pinch the metal together, and the resulting friction under the head compensates for the loss of tension area in the perforated plates.

To say that nothing can be done to avert the condition is to forget that rational methods of design can be reverted to that treat a perforated plate as just what it is—namely, a plate that is weakened by the holes punched in it, good only for its net section in tension.

The method once universally used of finding the effective depth of a plate girder and proportioning the compression flange for its gross area and the tension flange for its net area is eminently rational and results in design to which exception cannot be taken from any standpoint.

The authors state, "Once a ductile material is strained beyond the yield point and the load is released, residual stresses are set up which will be beneficial in resisting the next applied load." This does not agree with abundant experience in the use of chains and axles. Chains will stand loads beyond their yield point for so long; then they are likely to break. Chain users know this, and they anneal their chains at times. Axles stand, repeatedly, the load that may finally break them. There is such a thing as fatigue, and frequently breaks occur with a comparatively small number of repetitions (not up in the hundreds of thousands or any such number).

Some years ago, in Pittsburgh, Pa., a wrought-iron plate girder broke apart in the tension flange and dropped the locomotive that it had doubtless carried many times. The flange plates broke sharply, very evidently from fatigue.

The rule that designs a plate girder for gross section in the tension flange may result (where thick metal and many cover plates are used) in 4 or 5 sq in. of metal punched away in one hole. This represents a stress of 70 to 90 kips being imposed on friction under the head of a single rivet, and that rivet might be loose.

Plate-girder design needs careful scrutinizing from a rational standpoint.

WALTER H. WEISKOPF,[20] M. AM. SOC. C. E. (by letter).[20a]—The profession is indebted to the authors for an investigation of an important feature of structural engineering. For many years holes have been deducted in computing the strength of plate girders as a matter of routine.

With the authors' measurements there is little room for dispute. When one considers that the net section extends over such a small length of girder as compared to the length between the holes, it is not surprising that the holes, whether open or filled by bolts or rivets, have small effect on deflections.

[20] Cons. Engr. (Weiskopf & Pickworth), New York, N. Y.

[20a] Received by the Secretary December 2, 1939.

Engineers have recognized this for many years and have generally used the gross moment of inertia when computing deflections or stiffness coefficients for use in designing statically indeterminate structures. Likewise, it is obvious that the holes have a negligible effect in shifting the neutral axis from the center of gravity of the section. For the same reason, also, a practically negligible increase in flange stress due to the holes was discovered by the method the authors employed in measuring such flange stresses. Since this stress was computed from the strain in a 10-in. length of flange, it was obviously determined by the average of the strains in many differential lengths of gross section and a much smaller number of differential lengths of net section. The method thus failed to detect the very high concentrations of stress around the holes well known to investigators of this subject and mentioned, but not quantitatively given, by the authors in Fig. 5.

With the authors' conclusions, however, there is room for very clear disagreement. The authors state that nothing can be done to prevent the high stress concentrations around holes. Actually, these can be eliminated very simply in all but girders with very large flanges by eliminating the flange rivets, either by using welded cover plates or by employing a flange composed of a split beam tee and two angles.[21]

The writer also emphatically disagrees with the authors' recommendation of using the gross moment of inertia with the usual fiber stress. Frequently, the area of the holes is very considerable as compared with the remaining flange section. In heavy cramped work a common flange consists of 8-in. by 8-in. angles and 17-in. or 18-in. cover plates, with four lines of rivets through the covers. Often the stagger is so small that the net section traverses all four rivets so that the cover plate area is as much as 22% rivet holes. This is far too important an item to treat lightly, and it is improper engineering to dismiss the high stresses around these holes with the casual remark "and nothing can be done to prevent them."

The purpose of strength computations is to provide a safe and reasonably efficient design. An efficient design is one in which all parts of the structure have about the same margin of safety or safety factor. It is obvious that a riveted girder with high stress concentrations at the holes is definitely inferior to a plain rolled beam or to a girder of another type which has no such holes; and a balanced design requires that this feature be given due consideration.

One way of compensating for the detrimental effect of the holes is the practice, extensively used, of designing by the net section. Another simpler, and perhaps more practical, method which has been suggested is to base the design on the gross moment of inertia but to reduce the fiber stress according to some function of the ratio of net to gross flange.

C. D. WILLIAMS,[22] ESQ. (by letter).[22a]—Some interesting data are offered in this paper, although the conclusions may not be concurred in by all engineers. It should be noted that the stress intensities reported by the authors are based

[21] "A Comparative Analysis of Plate Girders," by Walter H. Weiskopf and John W. Pickworth, Members, Am. Soc. C. E., *Civil Engineering*, November, 1934, p. 585.

[22] Associate Prof., Head of Dept. of Structural Eng., Fenn College, Cleveland, Ohio.

[22a] Received by the Secretary December 12, 1939.

on average stresses as computed from the strains on 10-in. gage lengths. This set-up, of course, does not give the maximum stress values—a fact that is recognized by the authors.

It should be proper at this point to make some comparisons of their results with the results that would be expected when the stresses are computed according to the commonly accepted practice of deducting rivet holes in the tension flange.

The stretch of any finite length under stress is equal to the unit stress times the length, divided by the modulus of elasticity. When there are two or more unit stress intensities in the length considered, the stretch may be found by use of the average stress per unit length. If there were no reductions in area throughout the length the average stress and the maximum stress would ordinarily be equal. However, when there are local reductions in area, the average stress producing elongation will be increased thereby. The reduction in area and the length over which it occurs are both factors in the increase of stretch due to a given force since they influence the average stress over the finite length considered. In other words, the stretch of a finite length due to load is inversely proportional to the volume taking stress.

In considering the effect of a local reduction in area upon the deformation of a finite length longer than that which is reduced comparison may then be made between the reduced and unreduced volumes for the finite length.

TABLE 7.—COMPARISON OF OBSERVED AVERAGE STRESSES AND AVERAGE REDUCTION IN AREA

Section mark	Tension Flange Make-Up		Gross area, in square inches	Deductions			Percentage volume deduction*	Observed stress increase due to holes
	Angles	Plates		In square inches	In cubic inches	In cubic inches per inch of length		
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
(a) Specimen PT-1: 5-In. Rivet Spacing								
C-1	2	....	2.38	0.56	0.44	0.088	2.8	3.0
C-2	2	1	3.88	0.94	0.66	0.132	3.4	2.7
C-3	2	2	5.88	1.31	0.88	0.176	3.0	3.8
D-3	2	2	5.88	1.31	0.88	0.176	3.0	2.1
Average	....	....	....	....	....	....	3.05	2.90
(b) Specimen PT-2; 2.5-In. Rivet Spacing								
C-1	2	....	2.38	0.56	0.33	0.132	5.6	7.0
C-2	2	1	3.88	0.94	0.55	0.220	5.6	6.3
C-3	2	2	5.88	1.31	0.77	0.310	5.3	5.3
D-3	2	2	5.88	1.31	0.77	0.310	5.3	7.0
Average	....	....	....	....	....	....	5.45	7.15

* Or, the equivalent average area deduction.

Table 7 contains certain properties of the girders of series C and D taken from the details of the paper. These are the series that had holes in the tension flanges. Values in Column (5) are obtained by deducting two $\frac{3}{4}$-in. holes from

the horizontal legs of angles and cover plates and one-half of a $\frac{3}{4}$-in. hole through the two vertical legs. The volume deduction shown in Column (6) is computed for sections in Table 7(*a*) as the volume of metal removed by two $\frac{3}{4}$-in. holes through the horizontal legs and cover plates and one $\frac{3}{4}$-in. hole through the two vertical legs; for sections in Table 7(*b*) the holes in the vertical legs are spaced 5 in.; therefore only one-half of the volume of these holes is included.

The results listed in Columns (7) and (8) are the deductions in volume and the percentage deductions in volume due to rivet holes. Column (9) is computed from Columns (1) and (2) of Table 3 and is the percentage by which the values of Column (1) are less than those of Column (2). Column (9), then, is the percentage of increase in the average stress which the authors found due to rivet holes. Column (8) is the percentage reduction of volume in the gage length due to rivet holes; or, inversely, it is the percentage increase in average unit stress.

The comparison of the values in Columns (8) and (9) is remarkable, especially in view of the authors' claim to an accuracy of only within about 1.5 per cent.

The writer would like to suggest the following conclusions on the basis of the data submitted by the authors:

1. In view of the fact that the average stress intensities as found by measuring strains on 10-in. gage lines check so closely the average values found by deducting rivet holes from the tension flanges, it would seem to follow that the maximum intensities of stress are more correctly given by use of the net moment of inertia than by the gross moment of inertia.

2. Since the measurement of strains for the purpose of locating the neutral axis was made on 10-in. gage lines and did not register maximum intensities of stress, local deviations of the neutral axis could be present without detection. Since the measurements all indicate that the average location of the neutral axis is moved up by drilling holes in the bottom flange, there seems ample justification of the usual procedure of considering the neutral axis to be at the gravity axis of the net section.

3. Since the average location of the neutral axis is only slightly above the gravity axis of the gross section this axis may be used satisfactorily for the computation of the deflection of the girder, as may the moment of inertia of the gross section; but deflection due to average stress and maximum stress intensity should not be confused.

4. Since the use of the gross section in computing stress intensity is obviously opposed to conservatism, and since there are likely to be stress concentrations in the region of rivets not affecting the longitudinal deformations, it seems proper to use the net moment of inertia in the design of girders until a more conservative and exact method is found.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

TRANSIENT FLOOD PEAKS

Discussion

BY IVAN E. HOUK, M. AM. SOC. C. E.

IVAN E. HOUK,[12] M. AM. SOC. C. E. (by letter).[12a]—The author has contributed a novel treatment of an old subject. Cloudburst floods and the almost instantaneous surges which often accompany them have been described in American scientific literature for more than a century. However, so far as the writer knows, the author's comparison of such phenomena on the basis of the maximum cross-sectional area per square mile of catchment basin is new. Such a basis of comparison is logical and about as practicable as any that might be adopted, especially when the intense rates of rainfall and runoff occur near the head of a canyon and can reasonably be assumed to extend over the entire catchment area. When this is not the case, as often happens, it would seem better to base the comparison on the maximum cross-sectional area per square mile of zone of intense rainfall and resulting intense runoff.

It is always desirable, when at all feasible, to estimate the maximum rate of runoff from the contributing territory; but, unfortunately, flow estimates can seldom be made with satisfactory degrees of accuracy in cases of cloudburst surges like those described by the author. Uncertainties regarding roughness factors, surface slopes, acceleration slopes, velocity distribution, and effects of transported debris are so great that flow calculations are extremely questionable, even when the maximum cross-sectional areas can be determined accurately. For this reason, the publication of data on cross-sectional areas, together with contributing catchment areas, for cloudburst flood waves of unusual magnitude, is valuable to the hydraulic engineer. The reader, if he wishes, can make his own estimate of the average velocity at the location of measurement, and can then calculate the probable rate of discharge and corresponding probable rate of runoff from the contributing drainage area above the cross section.

NOTE.—This paper by Henry B. Lynch, M. Am. Soc. C. E., was published in November, 1939, *Proceedings*. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion of the paper.

[12] Senior Engr., Technical Investigations, Bureau of Reclamation, Denver, Colo.

[12a] Received by the Secretary November 30, 1939.

Cloudburst surges are not confined to mountainous regions. They occur in practically all sections of the United States. The writer has personally witnessed such phenomena in the gently rolling topography of Iowa and Ohio, as well as in the comparatively steep topography along the east edge of the Colorado Rockies. However, in mountainous territory, destructive flood waves are caused by lower rainfall intensities, due to the more favorable conditions for rapid runoff. The rates of precipitation which caused the flood waves described by the author would seldom, if ever, cause destructive flood waves in the comparatively flat topography of the central Mississippi River Valley. Furthermore, cloudburst surges in mountainous regions transport much heavier loads of sand, gravel, and boulders than surges of comparable magnitude in regions of gently rolling topography where rates of movement are necessarily somewhat lower. Cloudburst surges in any locality may be intensified greatly if the storm moves downstream at about the speed of the flood wave.

One of the first cloudbursts recorded in the United States was the storm that occurred at Catskill, N. Y., during the evening of July 26, 1819, when approximately 13 in. of rain fell in three hours. Although the most intense precipitation fell relatively close to the Hudson River, the flood waves from small tributary drainage areas were great enough to erode new channels at several locations.

Another storm of historic interest was the one that occurred in Delaware County, Pennsylvania, about 20 miles southwest of Philadelphia, during the afternoon of August 5, 1843. Cloudbursts that afternoon precipitated a total rainfall of approximately 16 in. in three hours and caused the highest local floods known in Delaware County since the year 1700 A.D. Flood waves, several feet high, with almost vertical fronts, passed down some of the small creek channels which drain that section of Pennsylvania.

The storm which centered in Des Moines County, Iowa, about 15 miles northwest of Burlington, between 2 and 4 a.m. on August 16, 1898, was similar to the aforementioned Pennsylvania storm. Field investigations by Maurice Ricker,[13] reported to the Iowa Academy of Science, showed that approximately 16 in. of rain fell in less than three hours over an area of about 50 sq miles. The resulting flood runoff destroyed trees 12 in. in diameter, moved large rocks many feet from their original locations, cut new stream channels in some places, and caused damages estimated at $100,000.

Since the beginning of the twentieth century, many cloudburst storms and resultant intense floods have been described in engineering literature. Such descriptions have come from practically all parts of the country. A few typical examples which may be cited include the Heppner flood of June 14, 1903, in Oregon; the Salt Creek flood of September 28, 1923, in Wyoming; the Devil's Creek flood of June 10, 1905, in Iowa; and the Cabin Creek flood of August 9, 1916, in West Virginia. More recent examples include the flood of June 3, 1935, at Mexico City, Mexico; the Bear Creek Canyon floods of July 8, 1933, and September 2, 1938, near Denver, Colo.; the Tehachapi Creek flood of Sep-

[13] "The August Cloud-Burst in Des Moines County," by Maurice Ricker, *Proceedings*, Iowa Academy of Science, Vol. 6, 1898.

tember 30, 1932, in Kern County, California; the Republican River flood of May 31, 1935, in Nebraska; and the floods of July 7 and July 8, 1935, in south central New York. Many others might be mentioned. Undoubtedly, many isolated storms of cloudburst nature have occurred in sparsely settled communities and have never been reported because the resulting flood flows did not greatly damage human life and property.

Since cloudburst surges are caused by cloudburst rainfall, the rates of flow and total runoff during such occurrences can always be reconciled with the rainfall, providing all pertinent facts are known. This is axiomatic. Difficulties in reconciling cloudburst runoff with cloudburst rainfall are due to uncertainties in data. During such phenomena all factors involved are so variable that sufficient accurate information can seldom be secured. Rates of rainfall, areas on which such rates of rainfall occur, rates of runoff, areas contributing such rates of runoff, and respective time data for both rainfall and runoff are all uncertain. In attempting to correlate runoff with rainfall during such storms, the actual area on which the intense precipitation takes place should be considered, rather than the total drainage area above the location of stream measurement.

During intense storms of cloudburst nature, one automatic rain gage does not tell the entire precipitation story, even in relatively small catchment basins of one square mile or less. During such storms, rain gages, only 100 ft apart, may show differences in precipitation of more than 100%. It is not only possible, but in the writer's opinion also highly probable, that the rates of rainfall which caused the surges described by the author were considerably higher in some parts of the catchment basins than those actually measured at the gages.

Careful studies of intense storms show that abrupt increases in rainfall and runoff occur in some part or parts of the storm area in practically all storms of cloudburst nature, although such conditions may not be accurately measured and scientifically reported. The writer believes that the combination of conditions necessary to the formation of surges is a common characteristic of such phenomena. Consequently, he does not consider it logical to classify the floods of southern California, northern Utah, and Cardens Bluff as a special type of cloudburst phenomena.

The engineering profession is indebted to the author for his clear presentation of valuable rainfall, runoff, and erosion data pertaining to the southern California floods of January 1, 1934.

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DISCUSSIONS

TREND IN HYDRAULIC TURBINE PRACTICE A SYMPOSIUM

Discussion

BY MESSRS. W. S. PARDOE, AND DONALD H. MATTERN

W. S. PARDOE,[21] M. AM. SOC. C. E. (by letter).[21a]—This Symposium is a concise and readable summary of turbine testing methods brought up-to-date. The only reason for entering this discussion is in connection with Professor Moody's step-up formula, mentioned by Mr. Davis.[12] The only difference between that formula and the one suggested by the writer is the difference between a fourth and a fifth root. The writer will state his reasons for suggesting the latter, or fifth, root.

Rankin's expression for the loss of head in pipes derived by dimensional analysis is

$$\Delta h = k \frac{L}{d^{3-n}} V^n \qquad (11)$$

The value of n for smooth brass pipes is usually given as 1.75 (some experiments by the writer, on 1.5-in. brass pipe, gave $n = 1.79$). The value of n for rough pipes approaches 2.0; and n, therefore, lies between 1.75 and 2.

Equation (11) may be rewritten in the familiar Darcy form as

$$\Delta h = K \left(\frac{w}{V \rho d} \right)^{2-n} \frac{L}{d} \frac{V^2}{2g} = f \frac{L}{d} \frac{V^2}{2g} \qquad (12)$$

If d is the only variable and $n = 1.75$, $\Delta h \propto \frac{1}{d^{0.25}}$ and Professor Moody's step-up formula follows; but who will contend that the inside of the prototype is of the same physical smoothness as the model? This is necessary, as Blasius' expression

$$f = \frac{0.3164}{\left(\frac{V \rho d}{w} \right)^{0.25}} \qquad (13)$$

NOTE.—This Symposium was published in November, 1939, *Proceedings*. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion of the papers.

[21] Prof., Hydr. Eng., Civ. Eng. Dept., Univ. of Pennsylvania, Philadelphia, Pa.

[21a] Received by the Secretary November 22, 1939.

[12] "The Propeller Type Turbine," by Lewis F. Moody, *Transactions*, Am. Soc. C. E., Vol. 89 (1926), p. 625.

was obtained for smooth pipes. Furthermore although the writer has the highest respect for the way in which the turbine manufacturers get the water through with very small losses, nevertheless there will be some small losses due to impact and sudden enlargements which will vary as V^2 and hence will not vary with d.

In plotting coefficients for venturi meters for the Fluid Meters Committee of the American Society of Mechanical Engineers from about one hundred weighed water tests of venturi meters varying from 1 in. to 16 in., the writer expressed the lost head between the main and throat as

$$\Delta h_2 = k \frac{V_2^2}{2g} \qquad (14)$$

and found the empirical expression,

$$k = \frac{0.045}{d_2^{0.23}} \qquad (15)$$

With the foregoing considerations in mind, the writer suggested the fifth root as more conservative than the fourth, which he thinks is too liberal to the manufacturers. An example will show the difference, using Fig. 16. The maximum efficiency $e_m = 83.6\%$. Then, by Equation (9):

$$e_p = 100 - (100 - e_m)\left(\frac{16}{200}\right)^{0.25} = 91.3\%$$

and,

$$e_p = 100 - (100 - e_m)\left(\frac{16}{200}\right)^{0.20} = 90.1\%$$

which is a difference of 1.2%, but on the safe side for both customer and manufacturer.

No amount of abstract reasoning can take all the variables into account. The turbine manufacturers should have, or will soon have, sufficient reliable data on which to base a satisfactory step-up formula. The step-up formulas for discharge and horsepower suggested by Mr. Rogers (Equations (10)) are subject to certain minor corrections and could be derived a little more rationally, thus, if H is total head and Δh is the lost head: $e = \frac{H - \Delta h}{H}$; $\Delta h = H(1 - e)$; and the useful head $= H - \Delta h = e\,H$. Therefore,

$$\frac{Q_p}{Q_m} = \left(\frac{e_p\,H_p}{e_m\,H_m}\right)^{0.5}\left(\frac{D_p}{D_m}\right)^2 \qquad (16)$$

and

$$\frac{P_p}{P_m} = \left(\frac{e_p\,H_p}{e_m\,H_m}\right)^{1.5}\left(\frac{D_p}{D_m}\right)^2 \qquad (17)$$

DONALD H. MATTERN,[22] ASSOC. M. AM. SOC. C. E. (by letter).[22a]—Symposiums, such as this one presented so capably by Messrs. Winter and Davis, should open up discussion upon present-day hydraulic turbine practice.

[22] Assoc. Civ. Engr., Tennessee Valley Authority, Knoxville, Tenn.

[22a] Received by the Secretary December 1, 1939.

Interest in refinements of design and in the checking of the different assumptions by test has been increasing, due not only to the economics of the situation but to the trend of technological thought toward research.

Design.—Mr. Winter has made some interesting comments upon the different phases of a hydroelectric development. Throughout the entire paper he emphasizes concentration upon refinements of design as a further means of reducing hydraulic losses in the turbine water passages. It should be pointed out, however, that often the cost of securing desired flow conditions may amount to more than the capitalized value of the benefits received. In a drive toward technical perfection, it might be quite easy to lose sight of the economics of the situation, and each case should be studied by itself and decisions made accordingly.

Examination of plans for the Bonneville intake, scroll case, and draft tube, as reproduced in engineering literature, gives one the impression of complexity and an implication of difficult construction. It would be interesting to learn the actual cost of intake, scroll case, and draft tube as compared with a more simple design. Differences in $h_f + h_v$ for the unit as built, and for the conventional setup as originally tested, would be of assistance to those who may have similar problems. That there appears to be some difference of opinion between designers as to the value of additional structures in the intake and scroll case is evidenced by Mr. Davis' comment that "Experience seems to indicate that the most efficient intake and scroll case is the one with the fewest piers and obstructions" (see second paragraph preceding heading "Efficiency and Power Step-Up").

It has not been proved that splitters improve the efficiency and performance of a draft tube. Some well-qualified turbine experts say that splitters make the unit perform more smoothly and also improve the efficiency. Other equally qualified engineers claim that splitters do not help the operation of their turbines. Both groups have test curves to prove their individual contentions. To the average engineer, therefore, it would appear that, until definite information is available, he should choose the draft tube recommended by the specific turbine manufacturer, but should look with question upon any design that will increase his costs.

Cavitation.—As stated by Mr. Davis, development of high-speed turbines has resulted in lower pressures within the water passages and has placed more emphasis upon a critical cavitation coefficient. Pitting can be decreased greatly if the runner is set low enough to secure a sigma value greater than that which is critical. The necessary extra excavation is not desirable, however, because of the high cost per unit length of draft tube required by the large discharges. Just how high the runner can be placed above tailwater, therefore, depends chiefly upon experiments made on model runners. By the use of reliable test data it is possible, safely, to raise the runner higher than would otherwise be permitted by the manufacturer. In other words, factors of safety can be anticipated more closely and the resultant savings passed on to the customer.

With regard to cavitation laboratories, Mr. Davis has emphasized the fact

that, originally, it appeared necessary to test the model under the same head conditions as exist in the field, and that later developments have shown this to be unnecessary for heads greater than 10 m. In tests conducted at Holtwood in 1938, the question arose, and different sigma curves were run at varying heads in order to check this point. These test heads varied as much as 10 ft at the same sigma point, but showed no difference in regard to the location of critical sigma. Still later tests, which were computed at the new Baldwin-Southwark cavitation laboratory under approximately 20-ft to 30-ft head, checked very closely with an homologous runner that had been tested at Holtwood under a much higher head. From these very meager data, it would appear that for practical test purposes Prof. E. W. Spannhake's upper limit could be lowered to approximately 7 m. Further data are desirable, and any assistance given by suitably equipped laboratories would be appreciated by the profession.

The curves shown in Fig. 16(*a*) illustrate possible cavitation test results. In them the efficiency, horsepower, and discharge breaks lie in a vertical line. The writer's experience indicates, however, that they are more ideal than typical, although it is true that breaks like those illustrated do occur at times. Performance at the point of critical sigma does not necessarily result in an increase in discharge, a decrease in power, and a resultant drop in efficiency. In tests conducted at two different laboratories, on two different units, it has been noted that there are cavitation curves where the discharge has decreased almost directly with the power. The result is that efficiency has dropped little, if any, at the lower sigma values. On other curves, the discharge remained practically a straight line with all the changes in efficiency characteristic being caused by variations in power. The question might be asked whether the discharge characteristic is not more dependent upon laboratory idiosyncrasies than upon other factors.

Breaks in the sigma-horsepower curves have been found to be much more consistent. Using these horsepower breaks, the doubt connected with a proper power step-up to the large unit is eliminated. Since cavitation is tied up with the difference of pressures between the two faces of each blade, limitations of unit generation based upon horsepower, stepped up from the corresponding break in the model curve according to the three-halves power of the head, appear to be more conservative than using the discharge break.

Modification of limits so determined should be made as a result of field observations extending over a period of time long enough to cover an operating head cycle. Because of the nature of the step-up from model to prototype, these field tests will normally lower the point of critical sigma.

Efficiency and Power Step-Up.—Mr. Davis states that prediction of prototype performance from model tests is of particular interest in connection with many of the low-head plants now being constructed. There is some doubt whether a good test can be obtained by either the Allen or the Gibson methods because distances from their intakes to the center line of units are so short. In many instances, the effective metering sections are less than 40 ft, or less than that required for an accurate water measurement.

Many hydraulic engineers believe that accurate results cannot be secured by current-meter tests without high costs. Mr. Davis' treatment of the methods used at Safe Harbor is interesting, but similar applications must be made successfully elsewhere before complete acceptance will be given. As a result, there is no satisfactory testing method available for use at these low-head projects.

Several acceptance tests of propeller-type turbines have been run upon completely homologous models, and acceptance or rejection of the prototype determined upon this basis. Probably the first such test conducted was made in July, 1935, upon a fixed-blade propeller turbine having a diameter of 16 in. and a scale ratio to the prototype of 1 : 16.5. The model efficiencies were stepped up by application of the Moody formula (Equation (9)) and compared with the guaranteed values. Final power guarantees, however, were based upon index tests conducted in the field.

With regard to step-up of efficiencies, it should be noted that the head factor given in the formula is not being used at the present time by Professor Moody or by several of the turbine manufacturers. A test witnessed by the writer is probably one of many which tends to show that this part of the equation can be eliminated. The model was first tested under heads from 8 to 10 ft and then under the 35 to 50 ft which exists at Holtwood. Under unit speed conditions, corresponding to normal operating heads, the efficiencies between the two laboratories checked within close limits.

Professor Moody's original presentation of a proposed step-up formula[12] did not specifically state how the method should be applied to test data. Different laboratories have been using the formula in different ways, and, although the peak efficiency for the best speed is the same, there is a variation at other parts of the performance range. An example showing effects of three methods of applying the formula to a Kaplan unit is shown in Fig. 18. For illustrative

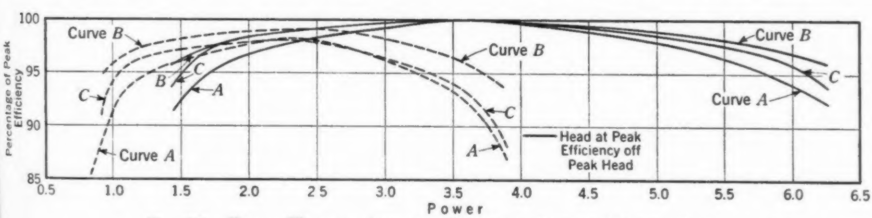

FIG. 18.—THREE WAYS OF APPLYING EQUATION (9) TO A KAPLAN UNIT

purposes, efficiencies are shown as percentages of the peak, and the power as arbitrary values. Curve *A* demonstrates the method recommended by Professor Moody, and was obtained by computing the efficiency increment for the turbine, irrespective of speed or blade angle, and adding the constant figure to the envelope efficiency for any head curve covered in the test. Curve *B* was secured by applying the Moody formula to each point of the model envelope. Curve *C* was obtained by computing the peak prototype efficiency for each

[12] "The Propeller Type Turbine," by Lewis F. Moody, *Transactions*, Am. Soc. C. E., Vol. 89 (1926), p. 625.

blade angle, and then finding the ratio of that value to the peak model efficiency. All points of that blade angle were multiplied by this factor to secure the prototype efficiency. The envelope curve for the unit was then drawn through these stepped-up values. Although there is not much difference at the head corresponding to the best turbine speed, the variation becomes more apparent at other heads. Curve *B* is always highest and flattest, curve *A* the lowest, and curve *C* is usually between the others.

It would be of value to the profession if the turbine manufacturers would cooperate not only in arriving at proper exponents to be applied to the ratio of runner diameters, as suggested by the author, but also in adopting the most nearly correct method with which to apply the formula to off-peak efficiencies. The manufacturers are the only ones who have information concerning the large number of installations which are needed to form the basis for such formula alterations.

Mr. Winter suggested another method of estimating the field performance when an accurate model test has been made, but failed to give any details of what he calls the "power step-up" method. Discussion of this method, together with several examples, would comprise a valuable contribution to the symposium.

Runaway Speed Tests.—Testing of model units for runaway speeds under different possible operating conditions is of considerable value. In 1935, J. D. Scoville showed[23] that runaway speed varies with sigma as well as with the usual factors of head and diameter of runner. To have a true picture of the overspeed conditions for which the generator should be designed, the speeds must be compared for the different sigmas that can occur under critical head conditions. Such model tests now form a part of several acceptance test programs, and, when conducted ahead of the purchase of the generators, may make it possible to buy the electrical equipment more economically.

Field Tests.—No discussion of field tests is complete without comment upon index methods of checking turbine performance. For practical purposes, it should be noted that a gate opening-output curve is sufficient to determine the best point at which to operate an individual unit. For more than one unit in a plant, and for Kaplan units, the relative efficiencies may be secured by including scroll-case differential pressure data with the output information.

Taps to indicate differential pressures may be placed either in the turbine speed ring columns, or in the scroll case itself. The latter type, in which one tap is placed in the outer wall of the scroll case and the other one in the casing just above the speed ring, is illustrated in Fig. 13, and is thought by many engineers to give more consistent results.

An attempt was made to coordinate model and field index tests at one plant by putting taps in the model at the same position as they were installed in the large unit. Care was exercised in locating the piezometer taps and in reading the water columns during the model tests, but discharges did not step up as would be expected for the prototype. Further research into correlation between model and prototype discharges might prove of value.

[23] "Cavitation Testing of Model Hydraulic Turbines and Its Bearing on Design and Operation," by L. M. Davis; discussion by J. D. Scoville, *Transactions*, A. S. M. E., Vol. 58, 1936, p. 323.

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DISCUSSIONS

FLASH-BOARD PINS

Discussion

BY JULIAN H. WHITE, ASSOC. M. AM. SOC. C. E.

JULIAN H. WHITE,[12] ASSOC. M. AM. SOC. C. E. (by letter).[12a]—The extensive tests reported by the authors should help to restore confidence in the use of pipe flash-board pins. Although there is ample evidence that, in the past, their behavior under overflow conditions has been rather unsatisfactory, the results shown in Table 3 indicate that, by the use of the data obtained, failure can be predicted with remarkable accuracy. For practical purposes, a variation of only 2% of the head, as obtained by the authors, would meet every requirement. However, if such close results are expected with conditions different from those under which the tests were made, a better understanding of some factors not evaluated in the paper is necessary. The reduction of pressure at the top of flash-boards, due to overflow, and the rise of water against the flash-boards beneath the nappe can cause an error in head at failure in excess of 2 per cent.

Through the courtesy of Charles M. Allen, M. Am. Soc. C. E., Director of the Alden Hydraulic Laboratory of the Worcester Polytechnic Institute, the writer made a few brief tests to obtain more information on these two factors, and the result may be of interest.

For the tests, a 5-ft glass-sided flume was narrowed to a width of 1 ft for a length of about 14 ft, and a wooden flash-board 1.51 ft high was set in the lower end. The back of the board was faced with a $\frac{1}{16}$-in. brass plate, set with the top above the wood to form a crest, and carefully drilled for 11 piezometers. The piezometers were set in the center of the board 0.010, 0.020, 0.040, 0.080, 0.161, 0.318, 0.450, 0.750, 0.950, 1.200, and 1.451 ft below the crest, and were connected to 0.5-in. gage glasses. Water, measured by venturi meter in a 12-in. pipe, passed through corrugated baffles before entering the flume. The head on the crest was measured by means of a piezometer set in the flume wall about 6 ft up stream from the flash-board, and in some tests was checked

NOTE.—This paper by Chilton A. Wright and Clifford A. Betts, Members, Am. Soc. C. E., was published in May, 1939, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: November, 1939, by Messrs. William P. Creager, Lincoln W. Ryder, and E. T. Schuleen; December, 1939, by John E. Field, M. Am. Soc. C. E.

[12] With Howard M. Turner, Cons. Engr., Boston, Mass.

[12a] Received by the Secretary November 28, 1939.

with a point gage. In certain runs, where a high velocity of approach was desired, the bottom of the channel was raised to a point 0.50 ft below the crest.

Measurements of pressure on the up-stream face of the flash-boards were made both with the boards vertical, and inclined down stream at an angle of 20° with the vertical. The heads ranged from 0.27 ft to about 1.20 ft, and the velocity head in the approach channel varied from 0.001 ft to about 0.13 ft. The results are shown in Fig. 16, in which the reduction in pressure for different points on the boards is expressed as a percentage of the total head on the crest. The tests showed that in the case with the boards vertical the pressure head at the bottom, and for some distance up, was greater than the depth by an amount very nearly equal to the velocity head. Therefore, the total head (that is, the measured head on the crest plus the velocity head due to the average velocity of approach) was used for the reference. With the inclined boards, the increase in pressure at the bottom was not so pronounced.

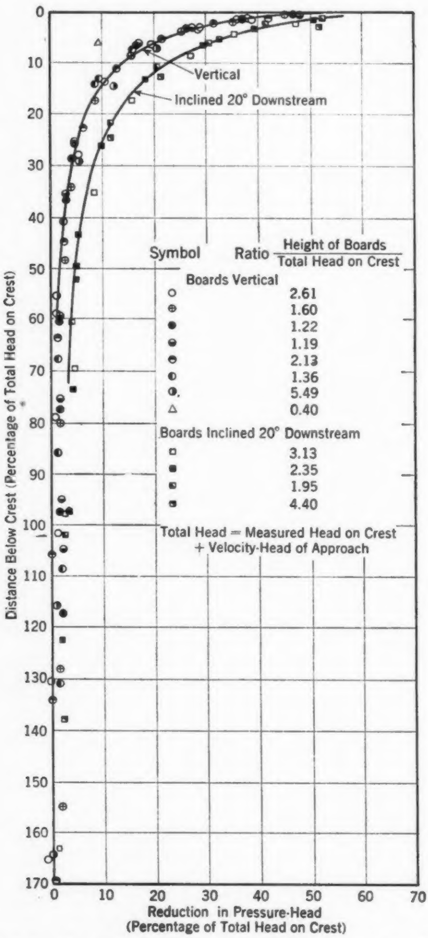

FIG. 16.—REDUCTION IN PRESSURE ON FACE OF FLASH-BOARDS WITH OVERFLOW

The fact that the pressure distribution is very nearly the same for a fairly wide range of heads indicates that, for the purpose of flash-board design, the reduction in pressure is a function of the total head on the crest only. Although this was not true for those tests with extremely high velocity of approach, such high velocities would not normally be met in practice. The pressure is reduced much more on a board inclined 20°, its approximate angle a failure, than when the board is vertical. The area affected in both cases extends from the crest downward a distance equal to about one half the head. Shortly below this point, the pressure is the same as for static loading, with the depth increased by the velocity head of approach. With heavy overflow, the percentage of area of the board affected becomes greater, and, because of its distance from the base, it may alter the bending moment considerably.

Observations of the height to which water behind the nappe rises were made with the flash-board in the vertical position only. For these tests, the three

lowest piezometers on the boards were reversed so that they measured pressure on the down-stream side. A staff gage was mounted on the down-stream face of the boards for direct measurement of the water height. The sides of the flume were made tight for some distance down stream, and provision was made so that the nappe could be ventilated or sealed as desired. A piezometer in the side of the flume just below the crest was connected to a U-tube partly filled with water to determine the amount of vacuum existing under the nappe at any time. Tests were made with the water falling upon the horizontal floor of the flume, and also upon an inclined floor set at two different angles. In all regular tests there was no effect of back-water. The water left the flume a short distance below the point where it struck the floor, and dropped to the wasteway.

Runs were made with ventilated and unventilated nappe. The flow, head on the crest, staff gage reading, and piezometric heights were recorded, as well as very approximate measurements of the nappe and the angle of the jet with the floor. In the unventilated runs, the vacuum in feet of water existing under the nappe was also measured.

It was noted that when the underside of the nappe was fully ventilated, and no vacuum existed, the water in the gages connected to the piezometers was all at the same elevation. Although the piezometers were set at different heights in the flash-boards, and the water under the nappe was somewhat agitated, the piezometer gages agreed with the staff gage. Thus it was concluded that when ventilated the height of the standing water is a measure of the pressure exerted against the flash-boards, and the distribution is the same as for the static condition.

A. Schoklitsch[13] points out that the rise of water behind the nappe of a weir is not due entirely to the existence of a partial vacuum, but is partly caused by the reaction of the water being turned to its final direction of departure as it strikes the floor. Those tests with the ventilated nappe were analyzed on this reaction basis, and were found to agree very well. The

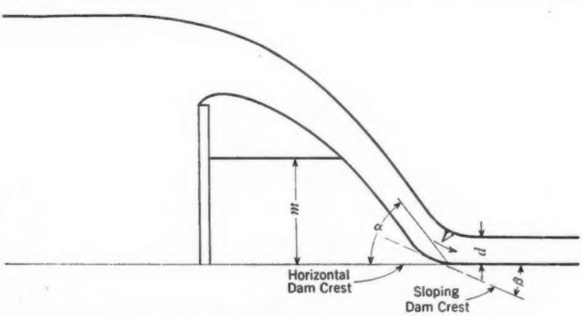

FIG. 17.—STANDING WATER BEHIND NAPPE

equation given by Schoklitsch indicates heights of the standing water greater than those found, so that another expression was used.

Referring to Fig. 17, the reaction of the water flowing at velocity V being turned through the angle α is balanced by the standing water behind the

[13] "Hydraulic Structures," by A. Schoklitsch, published by A. S. M. E., 1937.

nappe, or

$$\frac{w\,q}{g}\,(V - V\cos\alpha) = w\left(\frac{m^2}{2} - \frac{d^2}{2}\right)\ldots\ldots\ldots\ldots\ldots(23)$$

and

$$m = \sqrt{\frac{2\,q\,V}{g}\,(1 - \cos\alpha) + d^2}\ldots\ldots\ldots\ldots\ldots(24a)$$

Where the sloping floor made an angle β with the horizontal, the equation

$$m = \sqrt{\frac{2\,q\,V}{g}\,(\cos\beta - \cos\alpha) + d^2\cos^2\beta}\ldots\ldots\ldots\ldots(24b)$$

was used. The value of the depth d is quite uncertain because of the curved transition, and with a sloping floor it becomes even more indefinite. The term $d^2\cos^2\beta$ is only an approximation used because greater refinement seemed unnecessary in view of its small value in comparison with the other term under the radical. Equations (24) do not consider losses and certain other phenomena and are therefore not exact. Probably the water behind the nappe above the height d accelerates the falling sheet in the horizontal direction, changing both the angle α and the velocity V. If the true values of these could be determined, other expressions would, perhaps, give better results. For the method used in estimating the reaction height for any particular flash-board installation, and for the few experiments with which it was checked, the formulas given appeared applicable.

The heights of the standing water as observed in the tests under various conditions are shown in Table 8, Column (4). The heights as calculated by

TABLE 8.—Rise of Water Behind Nappe

Head on crest of boards, in feet	Fall from crest to floor, in feet	Angle of the floor with the horizontal	Height of the Standing Water Behind the Nappe Above the Floor, in Feet			Head on crest of boards, in feet	Fall from crest to floor, in feet	Angle of the floor with the horizontal	Height of the Standing Water Behind the Nappe Above the Floor, in Feet		
			Observed from test	Computed from Equations (24)					Observed from test	Computed from Equations (24)	
(1)	(2)	(3)	(4)	(5)*	(6)†	(1)	(2)	(3)	(4)	(5)*	(6)†
0.78	1.5	0°	0.94	0.91	0.94	0.36	0.79	17° 45′	0.37	0.43	0.44
0.61	1.5	0°	0.79	0.77	0.81	0.50	0.86	17° 45′	0.47	0.53	0.57
0.45	1.5	0°	0.61	0.60	0.62	0.62	0.93	17° 45′	0.56	0.58	0.67
1.15	1.5	0°	1.25	1.14	1.27	0.77	1.00	17° 45′	0.65	0.74	0.80
0.72	1.5	0°	0.88	0.87	0.89	1.08	1.22	17° 45′	0.92	0.92	1.07
0.44	1.16	6° 24′	0.54	0.55	0.57						
0.58	1.17	6° 24′	0.64	0.65	0.71						
0.68	1.18	6° 24′	0.71	0.76	0.81						
0.77	1.20	6° 24′	0.80	0.86	0.90						
1.15	1.25	6° 24′	1.05	1.05	1.21						

* Using measured nappe dimensions from test.
† Using scaled nappe dimensions from plot of theoretical nappe shape.

Equations (24) from measurements of the nappe made in the tests are shown in Column (5). For these calculations, the measured angle α was used, and the thickness of the nappe at the foot of the fall gave a value for d which was

also used to compute the velocity V from the known discharge. The agreement is very good in the tests where the floor was horizontal ($\beta = 0°$) and fair for the inclined floor.

In the case of a proposed flash-board installation, the thickness of the nappe at the foot of the fall, the angle of the jet at contact, and the distance of fall, if striking an inclined dam face, would not be known. To see what kind of approximation could be made in this circumstance, values of these unknown quantities for the conditions tested were scaled from a published plot of theoretic nappe shape. The resulting calculated heights of the water are shown in Table 8, Column (6). Although the agreement is not all that might be hoped for, it offers a method of estimating a factor which has been entirely neglected in many flash-board designs. As the moment varies approximately as the cube of the heads on either side of the boards, and as the water rises behind the nappe only a small part of the head on the up-stream side, the discrepancy of 20% shown in some of the computed figures would be much reduced.

The vacuum that may exist under an unventilated nappe can vary over a wide range. Apparently the water continually carries out small amounts of air, and if the fall is low the underside can become completely filled with water, provided there is no means of access for the outside air. The writer made too few experiments to learn just what the pressure conditions are under this circumstance.

When a partial vacuum exists under the nappe, the pressures which occur on the flash-boards are best illustrated by comparison with those obtaining when the nappe is fully ventilated. In the ventilated condition, a given flow caused the standing water to rise against the boards to a height of 0.79 ft, indicated both by the staff gage and the gages connected to the piezometers. With the same flow, but with the nappe sealed, a vacuum of 0.12 ft was observed. The jet was deflected inward approximately 6°, and the staff gage read 1.01 ft above the floor. Gages, connected to piezometers located 0.06 ft, 0.19 ft, and 0.56 ft from the floor, registered pressure heads of 0.95 ft, 0.91 ft, and 0.87 ft, respectively, above the floor. The gage to the upper piezometer differed from the staff gage by 0.14 ft, or about the height of vacuum in feet of water. The gages to the other two piezometers differed from the staff gage by amounts less than the vacuum, probably because of the effect of impact on the lower piezometers which are in a region where the water under the nappe is more disturbed. It is interesting to note that, although the partial vacuum decreases the pressure on the down-stream side of the flash-boards above the standing water, the total pressure due to the standing water itself is greater than it would be if the nappe were completely ventilated. There is still so much uncertainty regarding the question of vacuum that, in the case of a proposed installation, it would seem advisable to provide means to insure ventilation.

To apply the results of these experiments to a practical problem, the writer recalculated the bending moment at failure for Test No. 32, using the reduction in pressure given in Fig. 16. This happened to be the test with the greatest overflow, where the area affected by this reduction would be large. To be

sure that the concrete dam used to support the pins had no effect on the pressure distribution, several runs were made using the same flash-board as previously, but with a wooden dam similar to that used in Test No. 32, all built to scale. No difference in pressure was noted.

The total bending moment, including the moment due to the weight of the flash-boards as given in Appendix III, was 44 200 in-lb, which is about 9% less than that computed by the normal moment equation. This gives a modulus of rupture for this test of about 71 700 lb per sq in., which is closer to the recommended value of 70 000 than the value in Table 2(*a*).

If the boards of Test No. 32 had been installed on a flat-crested dam, the standing water behind the nappe would have risen to about 1.9 ft, and this would cause a further reduction of the moment of about 6 500 in-lb. The combined effect of the reduction in pressure on the up-stream face and the back pressure would thus alter the bending moment by about 22 per cent.

Fortunately, the relationship between bending moment and head on the boards is such that the effect of any error in moment on the water-surface elevation is greatly reduced. In many flash-board installations the change in head due to the two factors discussed will be very small; however, where it is important that flash-boards fail within a narrow range, both the reduction in pressure on the face of the boards and the rise of water under the nappe must be considered.